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# NEW METHOD USING BILINEAR TRANSFORMATION FOR PARAMETER IDENTIFICATION OF ANTICORROSION COATINGS

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**Abstract –** The paper presents a new method of anticorrosion coatings diagnostics using bilinear transformation. It is possible to identify the parameters of the equivalent circuit of the coating on the basis of object impedance measurement at a few, optimally selected measurement frequencies. The rules of optimal frequency selection are given. The number of frequencies is equal to the number of the equivalent circuit components. The developed identification algorithm enables continuous monitoring of the coating performance. The main advantage of the method is a few (more than 10) times shorter identification process compared to traditional identification technique based on impedance spectroscopy followed by impedance spectrum fitting using computer programs (e.g. LEVM or EQUIVCRT), which are utilising complex non-linear least squares (CNLS) method.

Keywords: parameter identification, bilinear transformation and anticorrosion coatings.

## 1. INTRODUCTION

One of the methods used in diagnosis of anticorrosion coatings is impedance spectroscopy [1]. The method is based on measurements of the coating impedance in a wide frequency range in order to find out the coating equivalent circuit. The fitting of an experimentally obtained impedance spectrum to a model response of the equivalent circuit uses the least squares method, implemented usually in computer programs [2, 3].

The above-presented approach can be used only in the laboratory, because in the very low frequencies range (<0,1Hz) the measurements are very onerous due to the very long time of measurement (many hours). Additionally, the instrumentation (e.g. from Solartron) is very expensive. For economic and safety reasons, it is necessary to characterise the state of the anticorrosion coating directly in the field for estimation of the protection renewal time. Due to this need, the authors have developed a new method in order to avoid disadvantages of the method of impedance spectrum fitting. The method uses properties of bilinear transformation [4], which represents the circuit function of a multiparameter model as a function of each individual parameter. Inverse bilinear transformation allows determining each component value on the basis of impedance measurement on single frequency.

## 2. THEORETICAL BACKGROUND OF THE METHOD

The diagnostics of anticorrosion protection is complicated due to the multi-element equivalent circuit of the anticorrosion coating (Fig. 1).

When the coating is new and there is no electrolyte penetration, resistance $R_p$ is of the order of several GΩ and capacity $C_c$ values are from a few tens to a few hundreds of pF, so they determine the resultant impedance of the equivalent circuit. During the exploitation of the coating, the barrier protection decreases and the electrolyte begins to penetrate the coating. This causes that the $R_p$ resistance decreases and the capacity $C_c$ increases due to the increasing dielectric constant.

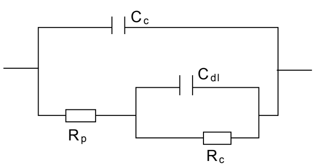



Fig. 1. Simplified equivalent circuit of the anticorrosion coating at the stage of electrolyte penetration and onset of the corrosion

where: $R_p$ – the resistance of the electrolyte in pores, $C_c$ – the capacitance of coating, $R_{ct}$ – the charge transfer resistance, $C_{dl}$ – the double layer capacitance.

Longer exploitation breaks the continuity of the coating and undercoating rusting appears, which is characterised by charge transfer resistance $R_{ct}$ and double layer capacity $C_{dl}$. During the stage before the onset of undercoating rusting and in its early stage (on a small area), $C_{dl}$ and $R_{ct}$ values are of the level of some nF and some GΩ, respectively.

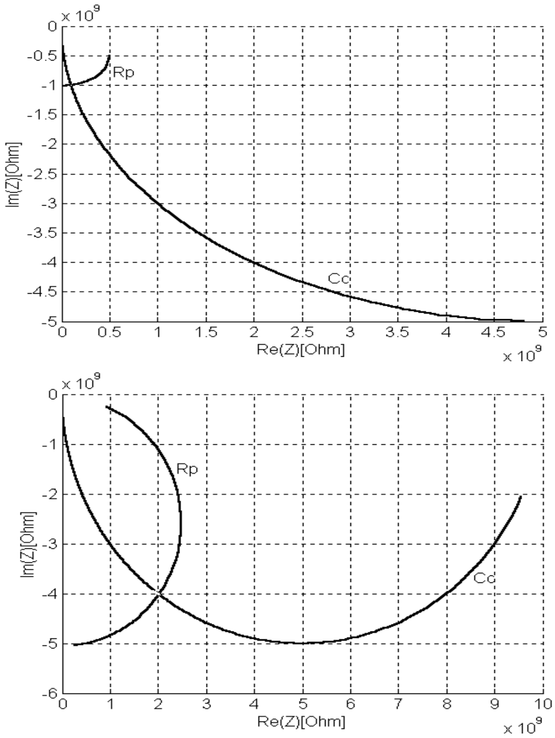
The impedance of the equivalent circuit (Fig. 1) can be expressed as a bilinear function of single element value $p_i$:

$$Z(j\omega, p_i) = \frac{A_i(j\omega)p_i + B_i(j\omega)}{C_i(j\omega)p_i + D_i(j\omega)} \qquad (1)$$

where: $A_i$, $B_i$, $C_i$, $D_i$ – complex coefficients,
$i$=1, 2, 3, 4 – *i-th* element of the equivalent circuit.
$p_i$=[$C_c$, $R_p$, $C_{dl}$, $R_{ct}$].

The presented function for all elements (e.g. for changes from 0,1 to 10 times nominal values) can be drawn on a complex plane as curves crossing at a point in which all elements have a nominal value. The nominal values of elements: $C_c$=330pF, $R_p$=10GΩ, $R_{ct}$=5GΩ, $C_{dl}$=20nF have been selected on the basis of [5] as representative for a new high thickness anticorrosion coating. An exemplary family of

$p_i$-loci at selected frequencies (0,5Hz and 0,1Hz) is shown in Fig. 2. As one can see, the shape of the curves depends on frequency. At 0,5Hz the $C_c$-curve dominates and at 0,1Hz the $R_p$ and $C_c$-curves are similar




a)

b)

Fig. 2. $P_i$-loci of bilinear transformation for a 4-element two-terminal network at a) 0,5Hz and b) 0,1Hz.

For the proposed diagnostic method, it is interesting to find out such measurement frequencies for which the impedance value depends mainly on a value of the element to be identified and the influence of other components is negligible. Such situation has been illustrated in Fig. 2a, where the impedance of a two terminal network mostly depends on the value of $C_c$. The $Z_i$ measurement at a selected optimal frequency ($\omega_i$) enables the identification of the *i-th* element value on the basis of inverse bilinear transformation of function (1) from (2):

$$p_i = \frac{D_i(j\omega_i)Z_i - B_i(j\omega_i)}{A_i(j\omega_i) - C_i(j\omega_i)Z_i}, \qquad (2)$$

although elements other than the *i-th* are known only approximately when $A_i \div D_i$ coefficients are calculated.

Analysing Fig. 2, it can be assumed that different optimal measurement frequencies exist for each element of the equivalent circuit of the coating. The frequencies are dependent on element values, so frequency estimation is possible for a new coating, the parameters of which are well known. In order to determine the optimal measurement frequencies, the relative sensitivity of the parameter under identification to other component changes has been used:

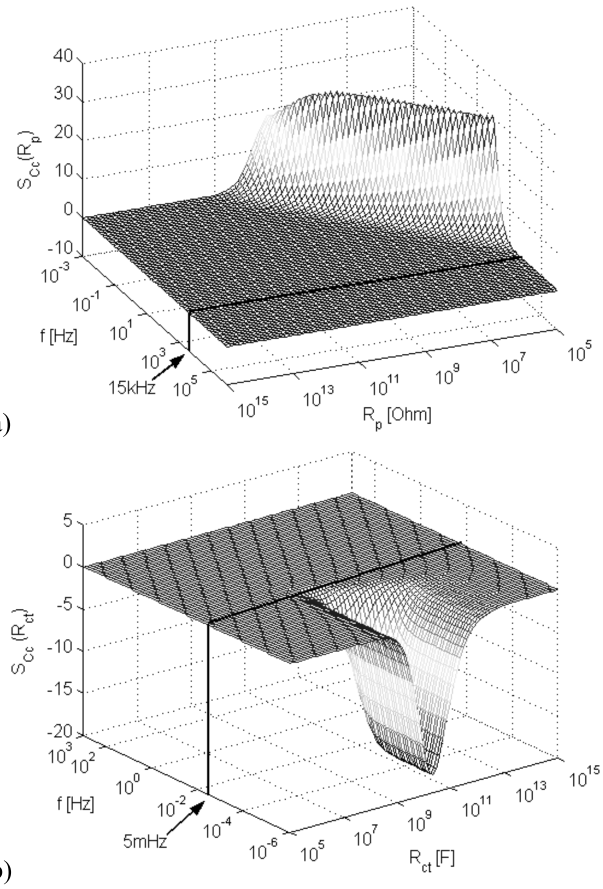
$$S_{p_i}(x_i) = \frac{\frac{\Delta p_i}{p_i}}{\frac{\Delta x_i}{x_i}}, \qquad (3)$$

where: $S_{p_i}(x_i)$ - relative sensitivity of parameter $p_i$ to changes of $x_i$,

$p_i = C_c, R_p, C_{dl}, R_{ct}$,

$x_i = C_c, R_p, C_{dl}, R_{ct}$ and $x_i \neq p_i$.

The values of the sensitivity have been estimated using numerical derivation of function (2), which allows determining of value of the component under identification. The results of exemplary calculation using Matlab have been shown in Fig. 3 and 4. The measurement frequencies are selected for each element separately. The sequence of the frequency selection is the same as the sequence of component identification.




a)

b)

Fig. 3. Relative sensitivity of $C_c$ element to a) $R_p$, b) $R_{ct}$.

At first the optimal measurement frequency $f_{Cc}$ is selected for identification of $C_c$ element, which is the least buried in the structure of the two terminal network. In Fig. 3a, the frequency limit (15kHz) has been marked. Above this value the sensitivity of the $C_c$ identification to $R_p$ changes is lower than specified threshold values (0,001). The relative sensitivity of the $C_c$ identification to $R_{ct}$ changes will be minimal when the measurement frequency will be greater than 5mHz (Fig. 3b). Similarly, the influence of element $C_{dl}$ can be minimised when the measurement will be made at frequencies greater than 14mHz. Taking into the consideration the three above conditions, it can be noted that the optimal measurement frequency for $C_c$ should be greater than 15kHz.

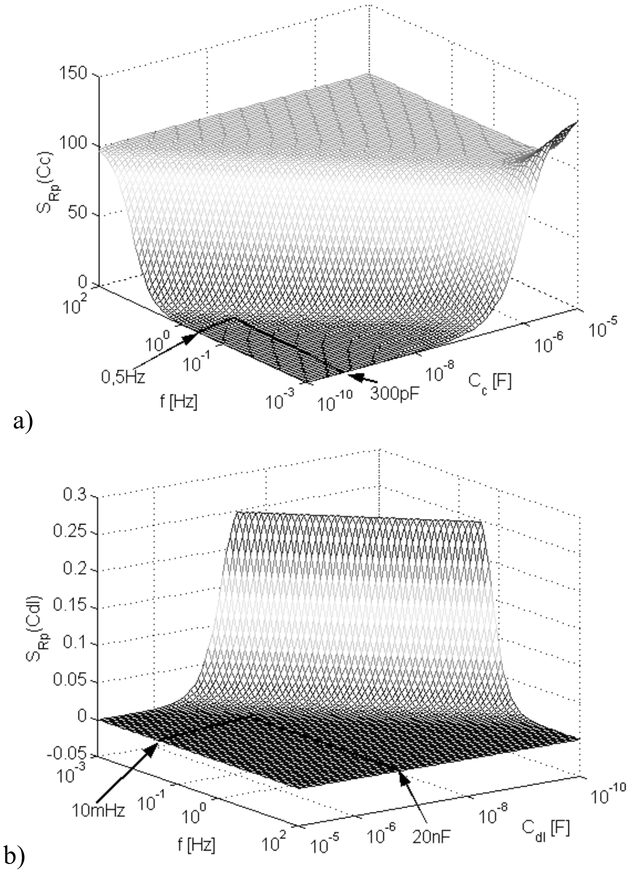



Fig. 4. Relative sensitivity of $R_p$ element to a) $C_c$, b) $C_{dl}$.

In the second step, the optimal measurement frequency $f_{Rp}$ for identification of element $R_p$ is determined. Fig. 4a and b present appropriate graphs of element $R_p$ sensitivity to other element changes. The capacity value $C_c$=330pF determined in the previous step has been marked in Fig. 4a. Due to this $C_c$ value, the optimal measurement frequency has to be lower than 0,5Hz. Element $C_{dl}$ (assuming its value is greater than 20nF) extorts a measurement frequency greater than 10mHz (Fig. 4b) and $R_{ct}$ (for approved changes in range of 100kΩ÷100TΩ) requires a measurement frequency greater than 0,2Hz.

Resuming, the optimal frequency for $R_p$ measurement should be determined on the basis of the following constraints: lower than 0,5Hz and greater than 0,2Hz, but the constraint on the side of 0,5Hz is stronger (due to greater sensitivity of $R_p$ to $C_c$ changes – Fig. 4a) so 0,2Hz is selected.

When selecting optimal frequencies for identification of other elements $C_{dl}$ and $R_{ct}$, the sensitivity of $C_{dl}$ element to $C_c$ and $R_p$ changes has been used (there is not dependence on $R_{ct}$) in the first case and the sensitivity of $R_{ct}$ to changes of $C_c$, $R_p$ and $C_{dl}$ in the second case. The changes of $C_c$ and $R_p$ have been limited to ±10% of the values determined in previous steps. The optimal frequency for measurement of element $C_{dl}$ was selected as 2,5mHz and for $R_{ct}$ – 1,4mHz, respectively.

The algorithm of the parametric identification of anticorrosion coatings has been developed on the basis of above discussion (Fig. 5).

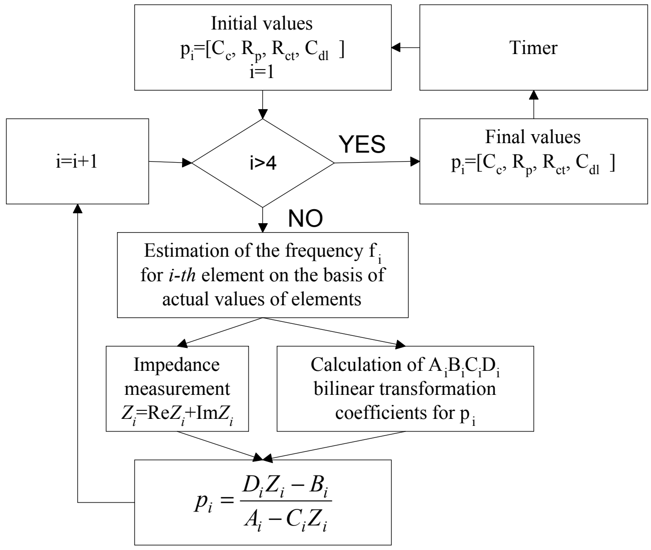



Fig. 5. The identification algorithm of anticorrosion coatings

## 3. THE IDENTIFICATION ALGORITHM

The elements are identified in a sequence depending on element placement in the structure of the equivalent circuit of the anticorrosion coating: $C_c$, $R_p$, $C_{dl}$, $R_{ct}$. In the algorithm the following steps can be distinguished:

1. At the before-test stage, the initial values of components of the two terminal network modelling real, good quality, anticorrosion coating ($C_c$=10pF, $R_p$=100GΩ, $R_{ct}$=100GΩ, $C_{dl}$=100pF) are assumed.
2. On the basis of sensitivity analysis, the optimal measurement frequency $f_{i=1}=f_{Cc}$ is determined. The real Re$Z_i$ and imaginary Im$Z_i$ parts of the coating impedance is measured at the selected frequency.
3. The coefficients $A_i$, $B_i$, $C_i$, $D_i$ of bilinear transformation are calculated for the first (i=1) element ($C_c$), next the value of $C_c$ is calculated according to (2).
4. Steps 2-3 are repeated at sequentially determined frequencies $f_{i=2}=f_{Rp}$, $f_{i=3}=f_{Cdl}$, $f_{i=4}=f_{Rct}$ in order to identify other elements: $R_p$, $C_{dl}$, $R_{ct}$.

Monitoring of the anticorrosion coating parameters requires repetition of the presented process. To do this, the algorithm is repeated from step 2 after a programmed delay time. The starting values of elements are replaced by values obtained in the last identification procedure.

To prove the usefulness of the method, simulation tests have been performed. To do this, 5 objects have been defined as two terminal networks based on the structure presented in Fig. 1 and values selected to model different stages of anticorrosion coating "life" (Table I).

TABLE I. Components values of objects under test

| Object | $R_p$ | $R_{ct}$ | $C_c$ | $C_{dl}$ |
|---|---|---|---|---|
| A | 100GΩ | 100GΩ | 10pF | 100pF |
| B | 10GΩ | 10GΩ | 100pF | 1nF |
| C | 10GΩ | 1GΩ | 1nF | 10nF |
| D | 1GΩ | 0,1GΩ | 1nF | 100nF |
| E | 1GΩ | 0,1GΩ | 1nF | 1μF |

The described two terminal networks have been identified using the developed method based on bilinear transformation (BIL) and using LEVM, program based on the well-known CNLS fitting algorithm [2]. For testing the objects, the impedance spectra has been generated in the way simulating real measurements, this means that a random component in the range of ±1% has been added to the impedance modulus values and a random component in the range of ±0,001° has been added to the impedance phase shift values.

For identification using the well-known CNLS fitting method, the measurements have been generated at frequencies in the range 1mHz÷20kHz assuming 3 points per decade (1-2-5). For the BIL method, the measurements have been simulated at frequencies determined by the described algorithm. Table II presents values of the frequencies for each object.

TABLE II. Optimal measurement frequencies determined by the BIL algorithm.

| Object | $f_{Rp}$ [Hz] | $f_{Rct}$ [Hz] | $f_{Cc}$ [Hz] | $f_{Cdl}$ [Hz] |
|---|---|---|---|---|
| A | 0,5 | 0,01 | 500 | 0,05 |
| B | 20 | 0,01 | 10000 | 0,4 |
| C | 20 | 0,01 | 10000 | 0,4 |
| D | 20 | 0,01 | 1000 | 0,4 |
| E | 20 | 0,01 | 1000 | 0,02 |

The identification results for both methods are placed in Table III.

TABLE III. The identification results for both methods.

| Object | Method | $R_p$ [GΩ] | $\delta_{Rp}$ [%] | $R_{ct}$ [GΩ] | $\delta_{Rct}$ [%] | $C_c$ [F] | $\delta_{Cc}$ [%] | $C_{dl}$ [F] | $\delta_{Cdl}$ [%] | Measurement time[s] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | CNLS | 100,4 | +0,4 | 100,3 | +0,3 | 9,99p | -0,1 | 100,6p | +0,6 | 1896,5 |
| | BIL | 100,2 | +0,2 | 99,9 | -0,1 | 10,01p | +0,1 | 100,5p | +0,5 | 121 |
| B | CNLS | 9,96 | -0,4 | 9,99 | -0,1 | 100,2p | +0,2 | 1007p | +0,7 | 1896,5 |
| | BIL | 10,1 | +1,0 | 10,1 | +1,0 | 100,1p | +0,1 | 1003p | +0,3 | 105 |
| C | CNLS | 10,14 | +1,4 | 0,84 | +16 | 1006p | +0,6 | 14,0n | +40 | 1896,5 |
| | BIL | 10,2 | +2,0 | 1,05 | +5,0 | 1001p | +0,1 | 10,2n | +2,0 | 105 |
| D | CNLS | 1,003 | +0,3 | 0,095 | -5,0 | 998p | -0,2 | 95n | -5,0 | 1896,5 |
| | BIL | 1,001 | +0,1 | 0,101 | +1,0 | 1004p | +0,4 | 100,8n | +0,8 | 111 |
| E | CNLS | 1,004 | +0,4 | 0,096 | -4,0 | 1001p | +0,1 | 1,12μ | +12 | 1896,5 |
| | BIL | 1,001 | +0,1 | 0,099 | -1,0 | 999p | -0,1 | 998n | -0,2 | 121 |

Analysing Table III, one can draw the following conclusions:

- The BIL method gives better results of identification with a maximal error lower than 1%, except object C, where $R_{ct}$ error of identification reaches 5%.
- The BIL method allows achieving a definitely shorter time of impedance measurement necessary to identify parameters of the object (max. time on the level of ca. 2min. in case of the BIL method compared to ca. 32min. in case of the CNLS method).
- Good properties of the BIL method have been achieved thanks to dynamically performed experiments – the measurement frequency is determined on-line, during each step of identification.

## 4. CONCLUSIONS

The developed method allows parametric identification of the equivalent circuit of an anticorrosion coating on the basis of impedance measurements at a few optimally selected frequencies, the number of which is equal to the number of identified elements. The method uses results from one measurement cycle as input data to another cycle, so it is dedicated for monitoring of coating performance.

The presented new method of parameter identification of anticorrosion coatings can be also used to the diagnosis of other objects. For example, in medicine, to detect pathological changes of skin, blood, and other physiological liquids, which can be described by multi-element two-terminal networks. The method can be also used to parameter estimation of impedance sensors (e.g. humidity sensors). In this case the estimation of only two parameters of a 7-element equivalent circuit is necessary to determine the relative humidity, so the identification process can be shortened to measurement at only two frequencies.

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