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HOMOTOPIC METHOD OF FAULT DIAGNOSIS OF PIECEWISE LINEAR CIRCUITS BASED ON SENSITIVITY MODEL

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Abstract – In the paper a new homotopic method of fault diagnosis of analogue piecewise linear (PWL) circuits based on a large-deviation sensitivity model is presented. Homotopy maps one function $f(x)$ into another $g(x)$ by changing the homotopy parameter $t \in [0,1]$. The idea of the method relies on using the function $f(x)$ to the description of the circuit under test (CUT) in the non-faulty state and the function $g(x)$ to the description of the faulty CUT. For verification of assumed fault hypotheses node homotopic paths are used.

The method enables localisation and identification of parametric (soft) faults in nonlinear circuits of PWL type. It is simpler in interpretation and calculation and more robust to the influence of CUT element tolerances and voltage errors in comparison with the homotopic method previously proposed by authors [8]. The method is illustrated by the example of single-fault diagnosis of a one-stage transistor amplifier.

Keywords: piecewise linear circuit diagnosis, sensitivity approach, homotopy method

1. INTRODUCTION

Fault diagnosis of nonlinear analogue electronic circuits is a difficult problem which is not solved generally till now. To simplify the problem, piecewise linear (PWL) models of nonlinear elements (Fig. 1) and circuits are used [1].

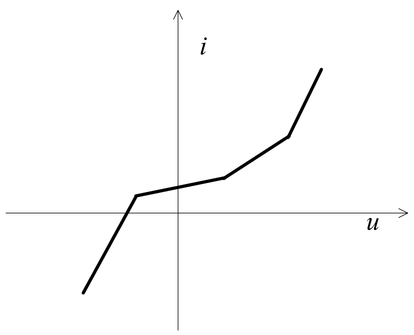

Fig. 1. Characteristic of piecewise linear model of a nonlinear element.

Recently, a verification technique for fault diagnosis of PWL circuits with limited measurement accessibility to internal nodes has been applied. This technique (developed for linear circuits) is based on the assumption of a hypothesis that some elements of the circuit under test (CUT) are faulty and the remaining ones are fault-free [4][5]. In the diagnostic procedure different hypotheses are assumed and verified. Each hypothesis is described by an appropriate diagnostic equation. Verification of the hypothesis relies on checking the consistency of this equation with measurement data [2][3].

The disadvantage of the verification technique in application to PWL circuit diagnosis is a large number of verified hypotheses and ability to diagnose only on the level of fault localisation. Authors proposed the application of a homotopy approach [6][7] to the verification technique of PWL circuit diagnosis, which overcomes disadvantages mentioned above. In the paper [8] we published a new verification method of PWL circuit diagnosis, based on homotopy, which sufficiently reduces the number of hypotheses. It also enables the localisation as well as identification of parametric faults.

In this paper we present another new homotopy method of PWL circuit diagnosis based on a large-deviation sensitivity model. The method is more clearer in interpretation, simpler in calculations and more robust on the tolerances influence of non-faulty elements as well as on voltage measurements errors.

In the first part of the paper, the basis of linear homotopy and idea of the method is explained. Next, the linear homotopy mapping is formulated, and principle of hypothesis verification with the aid of homotopic paths is explained. In the last part, the method is illustrated by an example of single-fault diagnosis of a one-stage transistor amplifier and compared with the homotopy method previously proposed by authors [8].

2. BASIS OF HOMOTOPY

To introduce the homotopy approach applied in the sensitivity-based method, first we explain a common definition of homotopy and next the properties of the linear homotopy mapping.

The homotopy is a continuous mapping of one function $f(x)$ into another function $g(x)$ as shown in Fig. 2, where the mapping parameter is a parameter t, called a homotopic parameter [8].

Many forms of the homotopy are known. One of the most known and useful in practice is linear homotopy

$$h(x,t) = (1-t)f(x) + t\cdot g(x) \tag{1}$$

with properties:

$h(x,t=0) = f(x)$ and $h(x,t=1) = g(x)$.

These properties can be utilized for simpler solving of the equation in the following way:

a) for $h(x,0)=f(x)=0$ there exists a solution $x(t=0)=x^0$, *easy to obtain*,
b) for $h(x,1)=g(x)=0$ there exists an unknown, *difficult to obtain*, solution $x(t=1)=x^*$,
c) *homotopic parameter* $t\in[0,1]$ enables to lead the continuous homotopic path $x(t)$ from x^0 to x^*.

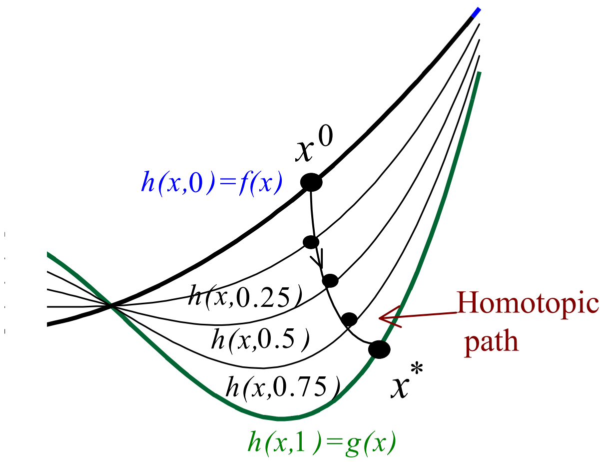

Fig. 2. The homotopy mapping of deformation of $f(x)$ function into function $g(x)$.

We use the linear homotopy as the base of a new verification method of PWL circuit fault diagnosis. The idea of the method (called further homotopic) relies on using the function $f(x)$ to the description of the CUT in non-faulty state and the function $g(x)$ to the description of the faulty CUT.

For verification of a fault hypothesis *node homotopic paths* are used associated with measurement circuit nodes and hypothetical faulty elements. If the assumed fault hypothesis is true the different node homotopic paths meet at the same end point and the location of this point identifies the parametric fault. If the hypothesis is not true they meet at different end points. In the case of a true hypothesis the location of the end path point identifies the parametric fault. Thus, the homotopic method gives the possibility of localisation as well as identification of parametric faults.

3. THE DIAGNOSTIC EQUATION

In the single linear region a PWL circuit can be described by bimultilinear functions dependent on many circuit parameters. For a single parameter it is possible to write these functions in bilinear form:

$$e_i\left(p_j\right) = T_i\,, \qquad T_i = \frac{n_{o,j} + n_{o,j}\,p_j}{d_{0,j} + d_{1,j}\,p_j}\,, \tag{2}$$

where e_i - measured node voltage in a region k; $i = 1,\ldots,m$, m - a number of nodes accessible for measurements; p_j - a circuit parameter in the region; $j = 1,\ldots,b$; b - a number of parameters; $n_{0,j}, n_{1,j}$, $d_{0,j}$ and $d_{1,j}$ - coefficients of bilinear function for p_j parameter in the region.

In each region the voltage change in the measurement node can be described as follows

$$\Delta e_i = e_i - e_i^0\,, \tag{3}$$

where e_i^0 - the node voltage of the non-faulty circuit in the region k.

Using the known large-deviation approach [9] the change of the node voltage in function of the parameter (a assumed single fault) p_j can be evaluated

$$\frac{\Delta e_i}{e_i^0} = \frac{S^{T_i}\dfrac{\Delta p_j}{p_j^0}}{1 + S^{D_i}\dfrac{\Delta p_j}{p_j^0}}\,, \tag{4}$$

where S^{T_i} - the small deviation sensitivity of the T_i, S^{D_i} - the small deviation sensitivity of the denominator of the T_i, Δp_j - the deviation of p_j from its nominal value in the region.

We use equation (4) after transforming it to the form

$$\frac{\Delta e_i}{e_i^0} - \frac{S^{T_i}\dfrac{\Delta p_j}{p_j^0}}{1 + S^{D_i}\dfrac{\Delta p_j}{p_j^0}} = 0\,, \tag{5}$$

as the basis of sensitivity approach to the homotopy method. From (5) we formulate functions $f(\cdot)$ and $g(\cdot)$ of the homotopy mapping. For the non-faulty circuit the function $f(x=\Delta p_j)$ takes the form

$$f(\Delta p_j) = -\frac{S^{T_i}\dfrac{\Delta p_j}{p_j^0}}{1 + S^{D_i}\dfrac{\Delta p_j}{p_j^0}} = 0\,. \tag{6}$$

For the faulty circuit, for which $\Delta p_j \neq 0$ and $\Delta e_i \neq 0$ one can get the function $g(x = \Delta p_j)$

$$g(\Delta p_j) = \frac{\Delta e_i}{e_i^0} - \frac{S^{T_i}\frac{\Delta p_j}{p_j^0}}{1 + S^{D_i}\frac{\Delta p_j}{p_j^0}} = 0 \cdot \tag{7}$$

From (6) and (7) in non-faulty and faulty states the linear homotopy in the region k in the measurement node i for the parameter p_j can be introduced

$$h(\Delta p_j, t) = (1-t)\left(-\frac{S^{T_i}\frac{\Delta p_j}{p_j^0}}{1 + S^{D_i}\frac{\Delta p_j}{p_j^0}}\right) - t\left(\frac{\Delta e_i}{e_i^0} - \frac{S^{T_i}\frac{\Delta p_j}{p_j^0}}{1 + S^{D_i}\frac{\Delta p_j}{p_j^0}}\right) = 0 \,. \tag{8}$$

Thus, on the basis of $h(\Delta p_j, t)$ we can define the linear homotopy for all regions and the node *i*, called further the generalized homotopy in the *i*-th node $H(\Delta p_j, t)$

$$H_i(\Delta p_j, t) = \begin{cases} h^1(\Delta p_j, t), & p_0^j \le p_j < p_1^j \\ \vdots & \\ h^k(\Delta p_j, t), & p_{k-1}^j \le p_j < p_k^j \\ \vdots & \\ h^K(\Delta p_j, t), & p_{K-1}^j \le p_j < p_K^j \end{cases} , \tag{9}$$

where $p_0^j, p_1^j, \ldots, p_K^j$ are boundary values of parameter p_j in regions, K – the number of linear regions.

From the generalized homotopy (9) we can obtain the node homotopic path leading from point zero (denoting the parameter in non-faulty state) to the deviation of the parameter Δp_j. The homotopic path in the single region k takes the form

$$\Delta p_j(t) = \frac{t\Delta e_i p_j^0}{e_i^0 S^{T_i} + t\Delta e_i S^{D_i}} \tag{10}$$

and the generalized homotopic path for all regions

$$\Delta P_j(t) = \begin{cases} \Delta p_j^{k=k_s}(t) \\ \vdots \\ \Delta p_j^k(t) & k = k_0, \ldots, k_f;\ k_0, k_f \le K \\ \vdots \\ \Delta p_j^{k=k_e}(t) \end{cases} \tag{11}$$

and it is called *the homotopic diagnostic equation*. For the assumed hypothesis p_j the algorithm has to be performed for all node measurement voltages. Thus, for one hypothesis, there can be *m* paths obtained with one start point zero and *m* end points. The length of the single path and the number of passing regions are different, depending on the fault. The homotopic paths $\Delta P_j(t)$ for several measurement voltages can be interpreted as traces from the non-faulty circuit, where parameter deviations are zero to the faulty circuit with some parameter deviations Δp_j.

4. HYPOTHESIS VERIFICATION

In the tolerance-less case, the verification of the hypothesis p_j relies on the comparison of several path ends. If all paths have the same end values $\Delta p_j^1, \Delta p_j^2, \ldots, \Delta p_j^m$ then the hypothesis associated with fault p_j is true with value p_j^f

$$p_j^f = p_j^0 + \Delta p_j = p_j^1 = p_j^2 = \ldots = p_j^m \,, \tag{12}$$

otherwise the hypothesis is false.

With regard to element tolerances and measurement errors, the simple comparison (12) is not enough. Thus, for verification, we apply the root-mean-square criterion

$$\delta(\Delta p_j) = \frac{\sqrt{\sum_{i=1}^{m}(\Delta p_j^i - \overline{\Delta p_j})^2}}{\overline{\Delta p_j}} \,, \tag{13}$$

where Δp_j^i - the path end for *i*-th node voltage and element parameter p_j.

The minimum value of $\delta(\Delta p_j)$ for all hypotheses

$$\delta_{\min} = \min(\delta(\Delta p_1), \ldots, \delta(\Delta p_b)) \tag{14}$$

specifies the most likely faulty element with value $p_j^f = \overline{p_j} = p_j^0 + \overline{\Delta p_j^*}$.

The graphical representation of the true hypothesis is shown in Fig. 3. All homotopic paths s_i (i=1,...,m) start from one nominal point 0 to several ends in the same region, which are at close range.

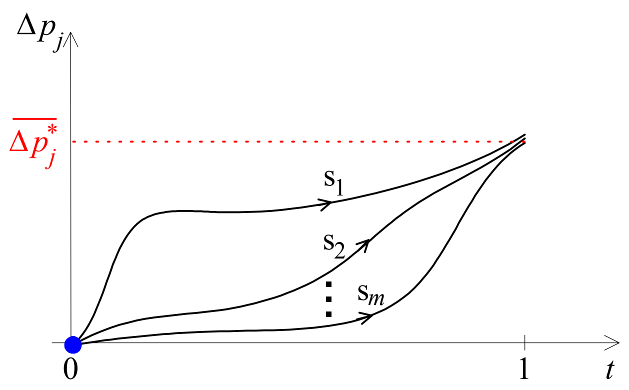

Fig. 3 Illustration of path ends lying at close range.

The maximal discrepancy between the mean value of all ends $\overline{\Delta p_j^*}$ and one from the end values is small. In this case all *m* paths passed from the nominal circuit region to the other, the same for all path ends.

In the case of a false hypothesis, the graphical interpretation is shown in Fig. 4. In the figure, starting from one nominal point 0, the ending points are at far distances from the mean value of $\overline{\Delta p_j^*}$.

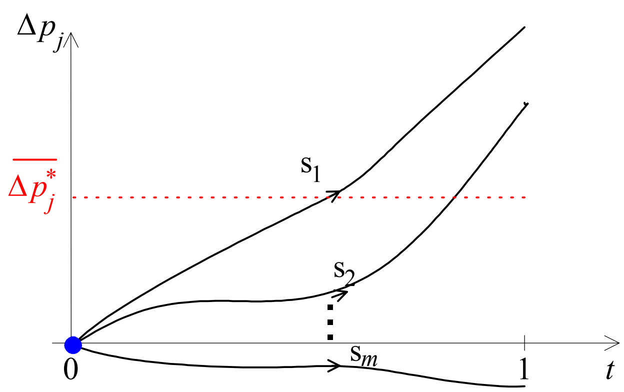

Fig. 4. Illustration of path ends lying in large distances.

Additionally, if path ends are in other regions, the fault hypothesis is false.

5. DIAGNOSTIC ALGORITHM

The fault identification algorithm for single faults can be illustrated by the block diagram shown in Fig. 5. The following steps of the algorithm can be distinguished:

1. Measure voltages in accessible nodes.
2. Formulate sets of diagnostic equations (11) associated with several hypotheses p_j.
3. For each hypothesis, find the values of path ends. If path ends for verified hypothesis are in different regions then the hypothesis is false and is not taken into consideration, otherwise from (13) calculate $\delta(\Delta p_j)$, $j=1,...,b$.
4. Find the minimum value $\delta_{\min}$. Hypothesis associated with $\delta_{\min} \to 0$ indicates the most likely faulty element.

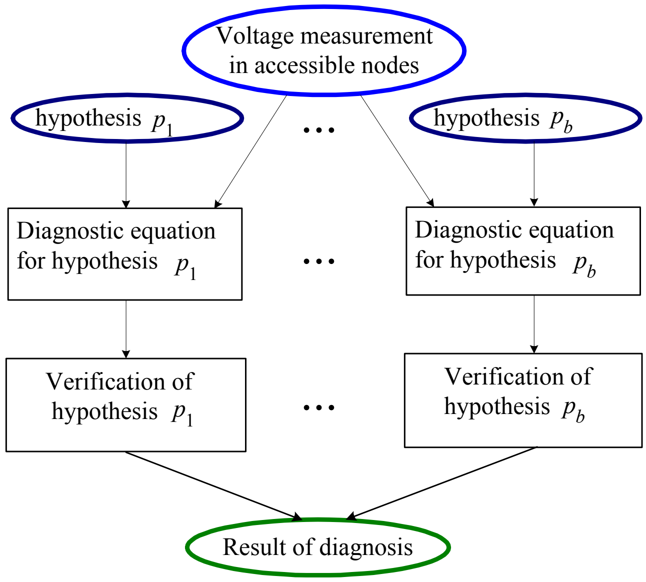

Fig. 5. Identification algorithm of single faults.

6. EXAMPLE

We illustrated the presented method on the example of the one-stage transistor amplifier shown in Fig. 6. The circuit contains *m*=3 nodes accessible for measurements: 1,2,4. The transistor model consists of two 7-segment PWL diodes (L_r=7). Thus, the overall number of regions is $K=(L_r)^2=49$.

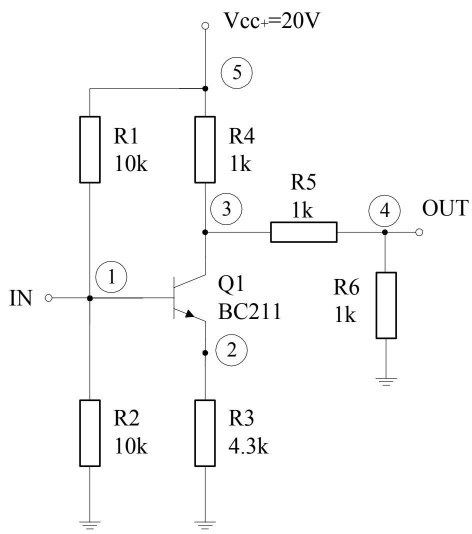

Fig. 6. One-stage transistor amplifier.

In this example we introduced the fault of the resistor R1, whose value we change from 10kΩ to $R1^*$=12kΩ. The verification results for some hypotheses p_j are shown in Table 1.

Let us analyze the hypothesis R1. Starting from the deviation ΔR1=0 one can get three path ends with similar values of R1 deviation with the mean deviation value $\overline{\Delta R1}^{*} = 2000{,}676\Omega$. The root-mean-square $\delta(\Delta R1)$ number is the smallest in comparison to the other $\delta(\Delta p_j)$. The identified fault is the resistor R1 with value $\overline{R1}^{*} = \overline{\Delta R1}^{*} + R1^{0} = 12000{,}676\Omega$. The relative identification error is 0,006% and the diagnosis time – 0,25s. The graphical illustration of the paths for ΔR1 is shown in Fig. 7. If t=1, the R1 values are almost the same for all nodes and node voltages are equal measurement values.

TABLE I The verification results for some hypotheses p_j in nominal circuit region k=43.

p_j	values for t=1 in region (7,1)			Result of verification $\delta(\Delta p_j)$
	e_1	e_2	e_4	
r_{d1}	-1000,34	21,453	20,001	false hypothesis – paths lead to different regions
α_F	0,900	0,870	0,910	0,039
R1	12000,001	12000,978	12001,050	0,0007
R3	321,043	405,074	4801,231	false hypothesis – paths lead to different regions
R4	-1988,591	-1978,290	964,743	3,568

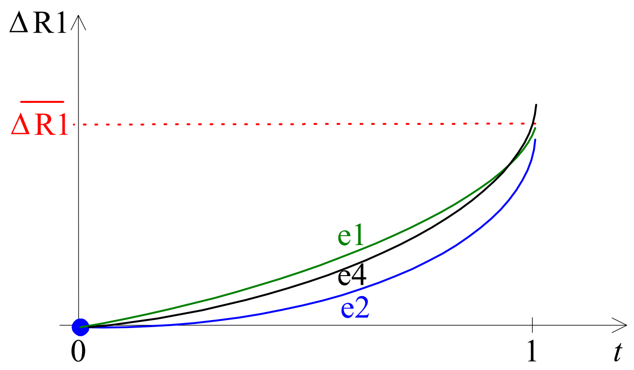

Fig. 7. Paths for true hypothesis of R1 fault.

For some hypotheses, several paths lead to different regions. It indicates that the assumed hypothesis is false. The example of his situation is analyzed for hypothesis p_j=R3. Paths for measurement node voltage e_1 and e_2 have the ends in region k=49. The values of R3 are similar. For the path associated with e_4, the path end is in the nominal region and R3 value is different from two previous values. On the basis of the information that the path ends are in different regions we find that hypothesis is false. Additionally, the path ends are in large distances from each other.

The graphical illustration of paths for the R3 deviation is shown in Fig. 8.

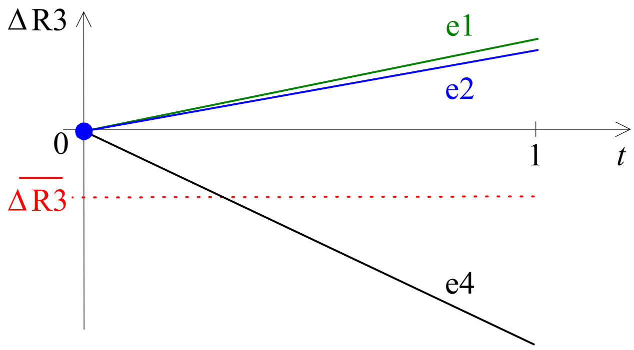

Fig. 8. Paths for false hypothesis of R3 fault.

7. COMPARISON OF THE HOMOTOPY METHODS BASED ON NODE CIRCUIT FUNCTIONS AND SENSITIVITY FUNCTIONS

As comparable criteria for both verification methods (presented here based on the sensitivity model [9] and proposed previously based on node circuit functions [8]) we choose a number of verified hypotheses, time of diagnostic process and complexity of homotopic paths treated as diagnostic equations.

The most substantial advantage of the homotopic methods is the small number of hypothesis verifications, for single fault diagnosis equal to the number of elements. The number of hypotheses for the both methods is the same. So, the diagnostic time is similar. It is little shorter for the sensitivity based method, because of the simpler form of the homotopic path (requiring less calculation in the after test stage) for this method in relation to the method based on the circuit function in such form

$$p_j(t) = -\frac{(e_i^0 + \Delta e_i \cdot t)\cdot d_{0,1} - n_{0,j}}{(e_i^0 + \Delta e_i \cdot t)\cdot d_{1,j} - n_{1,j}} \qquad (15)$$

Analyzing equations (10) and (15) we can see that in the sensitivity method we calculate the deviation of the parameter. The most important advantage of the sensitivity method is the robustness to the influence of CUT element tolerances and voltage measurement errors.

Additionally, the sensitivity equation has a clear physical interpretation, because it contains only physical measurable quantities. Next, the start point of the homotopic path is always at point zero.

In the circuit equation method we always start from the other point equal to the value of the parameter with nominal value. The equation contains parameters which do not have physical interpretation.

The sensitivity model seems to be the best one from our point of view as the basis of homotopic methods of PWL circuit fault diagnosis.

6. CONCLUSIONS

We presented the new homotopic method of fault diagnosis of analogue PWL electronic circuits based on the large-deviation sensitivity model of the CUT. In our opinion the sensitivity model is the best one for using in homotopy based methods of fault diagnosis of PWL circuits.

The advantage of the presented method in comparison with the previously proposed homotopic method using the node circuit functions is better conceptual and physical clearness and more robustness to the influence of tolerances and node voltage measurement errors.

The method is presented on the example of a single fault diagnosis, but it can be extended to double and multiple fault diagnosis. Under investigation is the double-fault version of this method.

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