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SPATIAL CURRENT DENSITY MEASUREMENT

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Abstract: The methodology of current density measurement in space plasma and the instrument for its realization - Split Langmuir Probe (SLP) - are discussed. An analytic approach to the investigation of SLP interaction with the surrounding plasma allowed to determine the conditions of its application and expected transfer function. Experimental evidence of SLP application efficiency is given.
Keywords: Split Langmuir probe, current density, measurement

1 INTRODUCTION

One of the most efficient tools of space plasma investigation is the study of electromagnetic fields of different nature and conditions of their generation and propagation. The main tasks of these studies is the determination of dispersion relations between the wave vector and the frequency. It was shown that simultaneous measurements of magnetic field and current density fluctuations allow to determine experimentally wave vector components [1]. The measurements of the magnetic field fluctuations usually are made by a variety of magnetometers using well developed methods. Unfortunately, up to the moment there are no reliable measurements of space current density, although some more or less successful attempts were made [1, 2]. In both these cases an instrument called Split Langmuire Probe (SLP) was used. It consists of two parallel conductive plates with surfaces spaced by as small as possible distance *d* and loaded by the resistor R_s (Fig.1). The SLP output signal U is formed as follows:

$$U = JSZ = IZ \quad , \qquad (1)$$

where J is the density of spatial current, perpendicular to the plate, I - current via SLP, Z is equivalent impedance of SLP preamplifier circuit. The SLP output is connected with the preamplifier and all arrangement can be presented by the equivalent circuit (Fig.2), where R_s, C_s are equivalent resistance and capacitance of the sensor and R_i, C_i - those of the amplifier.

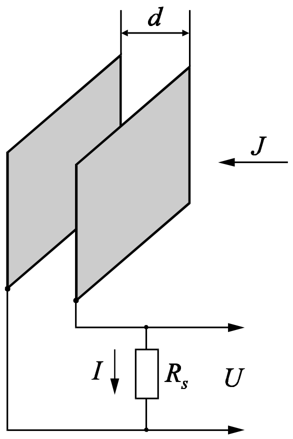

Figure 1. Split Langmuir probe diagram.

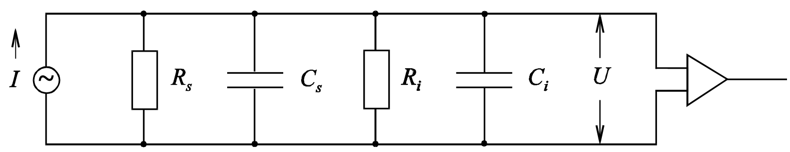

Figure 2. SLP equivalent diagram.

2 THEORETICAL APPROACH

The connection of U with J is investigated with following condition imposed:

$$d \le \lambda_d , \tag{2}$$

where λ_d is Debye's radius in plasma.

The relation U with J can be written as follows:

$$U = JSR \left(1 + (\omega\tau)^2\right)^{-0.5} \exp\left(i\varphi\right) \tag{3}$$

where R, τ - equivalent resistance and time constant of the circuit on Fig. 2,

$$R = R_i \left(1 + R_i R_s^{-1}\right)^{-1}; \quad \tau = CR ; \quad C = C_s + C_i ; \quad \varphi = -\operatorname{arctg}\left(\omega\tau\right).$$

Our aim is to study the dependence of the values

$$R_S = (\sigma_e S d^{-1})^{-1} \tag{4}$$

$$C_S = \varepsilon_e S d^{-1} \tag{5}$$

from the spatial plasma parameters which influence the values of dielectric permeability ε_e and specific electric conductivity σ_e of the plasma:

$$\varepsilon_e = \varepsilon_0 - n_e e^2 (m_e(\omega^2 + \nu^2))^{-1}, \tag{6}$$

$$\sigma_e = n_e e^2 \nu (m_e(\omega^2 + \nu^2))^{-1}, \tag{7}$$

$$\varepsilon = \varepsilon_e - j\sigma_e\omega^{-1}, \tag{8}$$

where ε - complex dielectric permeability of the plasma, m_e - electron mass and ν - electron-ion collision frequency.

From (3) transformation coefficient K_T is equal to:

$$K_T = UJ^{-1} = K_S \left(1 + R_i R_S^{-1}\right)^{-1} \left(1 + (\omega\tau)^2\right)^{-0.5} \exp\left(i\varphi\right) \tag{9}$$

and $K_S = SR_i$.

For K_T to be independent on plasma parameters and ω it is necessary to require the simultaneous fulfillment of two following conditions:

$$\left.\begin{aligned} R &= R_i \\ \left(\text{ùô}\right)^2 &= 0 \end{aligned}\right\} \tag{10}$$

If to substitute the terms in the expression (3) by their values and to take into account that [3]:

$$J = j\omega\varepsilon E , \tag{11}$$

it is possible to see that when conditions (10) are not accomplished the voltage *U* will be dependent on the *E d* value. These relations can be written approximately as

$$\left.\begin{aligned} N_1 R_1 &\le R_S \\ N_2 f &\le (2\pi|\tau|)^{-1} \end{aligned}\right\}, \tag{12}$$

where N_1, N_2 some factors, influencing the error δ_1 reflecting deviation of real relations (12) from ideal ones (10) according to the table 1 below.

Table 1. Error estimation

N_1	2.4	8.5	9.3	18.5	19.7	32.5	50.5	98.5
N_2	1	2	2,1	3	3,1	4	5	7
δ_1 (%)	29.3	10.6	9.7	5.1	4.8	2.99	1.9	1.0

From the conditions (12) it is possible to find the limitations on the operational frequency band of the system presented on Fig. 2.

The parameters f_c - critical SLP frequency - and f_0 – quasi-resonance SLP frequency - are introduced, which are influencing R_s and C_s values::

$$\left.\begin{aligned} f_c &= d\left(2\pi\varepsilon_0 K_S\right)^{-1} \\ f_0 &= \left(1 + C_i C_{0,S}^{-1}\right)^{-0.5} f_{pe} \end{aligned}\right\}, \tag{13}$$

where C_0 is the capacitance of SLP in vacuum, f_{pe} - Langmuir frequency of electrons in plasma equal to

$$f_{p,e} = (2\pi)^{-1} e(n_e(\varepsilon_0 m_e)^{-1})^{0.5}. \tag{14}$$

Taking into account that C_i and C_0 are very close one to other and for ionospheric plasma $f_{pe} >> \nu$, the following relation for the estimation of SLP operational frequency band can be obtained:

$$\left.\begin{aligned} f \geq (\nu_n f_{c,1}^{-1})^{0.5} f_{p,e} \\ \pm f^3 + f_{c,2}(1 + C_1 C_{0,S}^{-1})^{-1} f^2 - (\pm f_0^2 f) + \\ + N_2^{-1}\nu_n(1 + f_c \nu_n f_{p,e}^{-2}) f_0^2 \geq 0. \end{aligned}\right\}, \tag{15}$$

where $\nu_n = (2\pi)^{-1}\nu$, f_{c1} and f_{c2} - upper and lower SLP critical frequencies, determined by the given error of current-voltage transformation:

$$f_{c,l} = N_l^{-1} f_c, \; l = 1,2 \,. \tag{16}$$

Taking into account that R_i has to satisfy the relation

$$R_i = nkT \; \Delta f \left(J_N S\right)^{-2}, \tag{17}$$

where n, Δf - noise factor and amplifier's bandwidth, k - Boltzmann's constant, T - temperature, J_N - noise current density, the critical frequency can be calculated as follows:

$$f_c = d(J_N r)^2 (2\varepsilon_0 n \kappa T \Delta f)^{-1} \approx 4.1 \cdot 10^7 d(J_N r)^2 (nT \; \Delta f)^{-1}, \tag{18}$$

where r -radius of one round SLP plate.

For lower frequency band the satisfactory SLP operation can be achieved with the condition

$$f_c \geq f_{p,e}^2 \nu_n^{-1}. \tag{19}$$

3 MODELLING AND EXPERIMENTAL RESULTS

The dependence of normalized transformation factor $K_n = KK_s^{-1}$ module and phase on the frequency is given on Fig. 3.

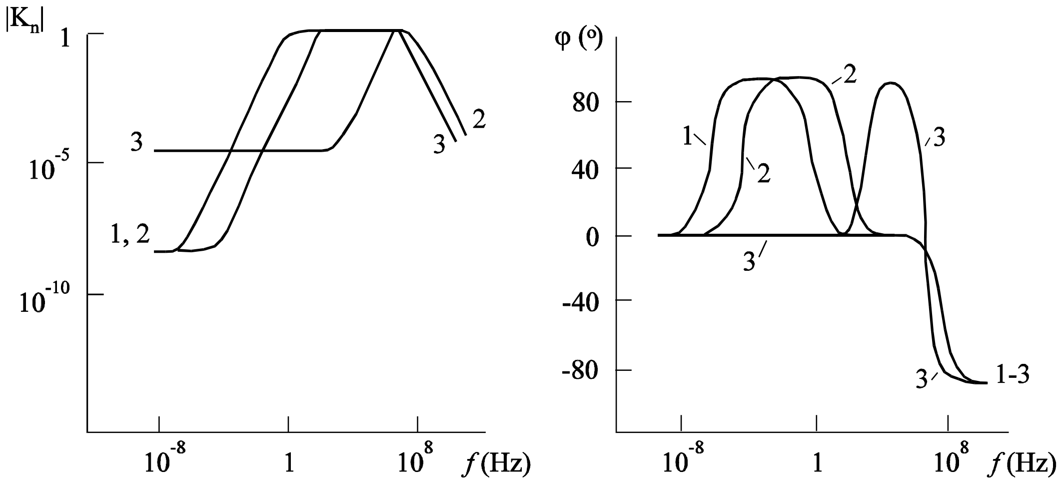

Figure 3. Module and phase of normalized transformation factor against frequency.

The numbers of curves correspond to the cases:

1) $f_{pe} = 10^4$ Hz, $\nu = 2.5 \cdot 10^{-6}\ s^{-1}$, $r = 0.2$ m, $d = 4$ mm, $J_n = 10^{-13}\ A \cdot \text{cm}^{-2}$, $\Delta f = 10$ Hz, $T = 300$ K, $n = 3$, $C_i = 5$ pF;
2) $f_{pe} = 10^5$, other parameters are the same as in case 1;
3) $f_{pe} = 10^7$ Hz, $d = 1$ mm, other parameters are the same as in case 1.

The correctness of the calculations carried out and the efficiency of the proposed methodology of the current density measurement in space plasma with the help of SLP were confirmed by the experimental data obtained on PROGNOZ-10 satellite.

Fig. 4 represents a fragment of magnetic field *B* and electrical current density *J* fluctuations obtained under the Earth's shock wave crossing. The high degree of correlation of these fluctuations is obvious which attains, according to calculations, 80%. This can serve the qualitative proof of validity of electric current fluctuations measurement in tenuous plasma with the help of SLP when Debye length λ_D is much larger than the probe size.

The quantitative estimation of such measurements correctness may be obtained by calculating from the experimental data the dispersion relation $k(\omega)$ in VLF-range and comparing it with the MGD-theory deductions.

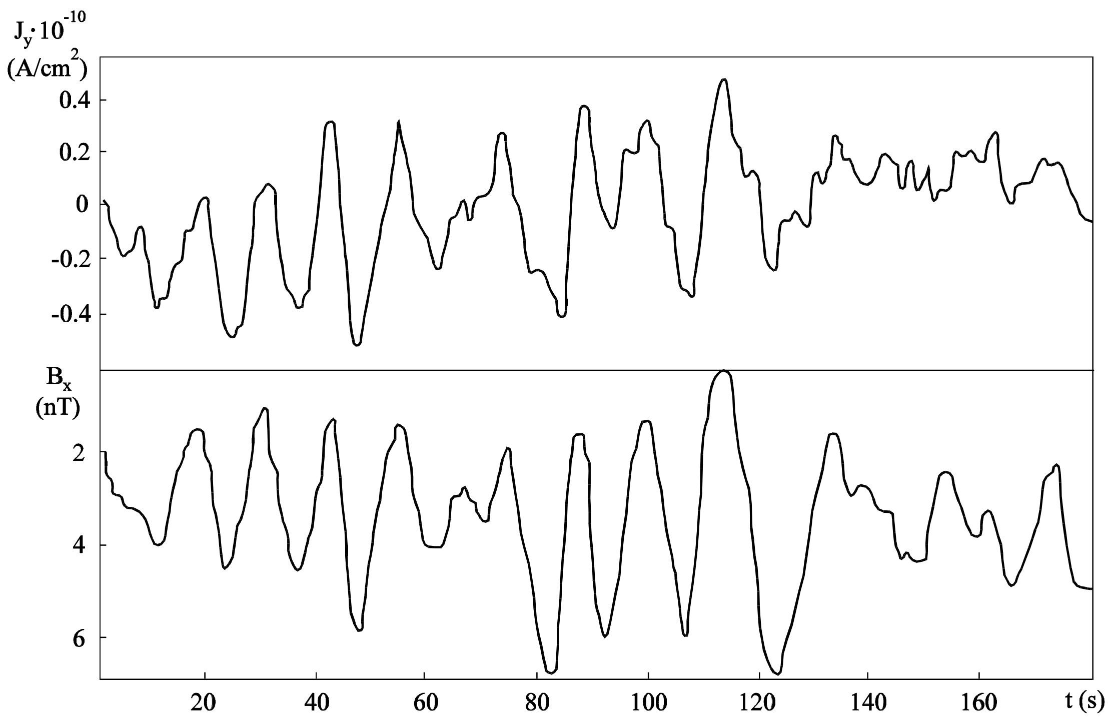

Figure 4. Magnetic field B and electric current density J fluctuations at the Earth's shock wave crossing.

A comparative analysis of experimental and theoretic dispersion relations requires the determination of the propagation direction of VLF waves and transmission to the stationary plasma frame of reference. Such an analysis was executed in the paper [4] where experimental data were compared with the linear two-fluids model of hot plasma [5]. A satisfactory agreement of theory with experiment was obtained: the distinction of location of experimental dispersion curves from the theoretical ones on the $(k \times \omega)$ plane by one of the axes does not exceed the factor of 2. But this estimation is the upper one of the error of current density measurements since the discrepancy of experiment and theory may be explained also by an approximation made of the linear model used for phenomena in the bow shock region description. In some cases of shock wave crossing such smaller discrepancy of experimental and calculated data, not exceeded 10%, was obtained [1].

All that said above proves the efficiency of the proposed methodological aspect of spatial current measurement and validity of the method for plasma phenomena investigation.

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