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# DYNAMIC FORCE MEASUREMENT IN PRACTICAL APPLICATIONS

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*Abstract: The problems involved in the measurement of dynamic forces in practical applications are discussed and methods for the reduction of the measurement uncertainty described. It is shown that force transducers with well-known dynamic properties must be used, calibrated by dynamic procedures, and it is demonstrated that in many applications the vibration behaviour of the whole mechanical structure must be analysed and taken into account.*
*Keywords: dynamic force measurement*

## 1 INTRODUCTION

In a great variety of applications, force measuring devices are used for the measurement of dynamic forces. A reduction of the measurement uncertainties requires that calibrated force transducers with well-known dynamic properties be used. In the past, dynamic calibration procedures were therefore developed at PTB to determine the dynamic response of force transducers [1,2]. Nevertheless, in special applications the arrangement of the force transducer, the mounting conditions and the whole mechanical structure of the measuring arrangement may significantly influence the uncertainty of dynamic force measurement in these particular cases. It is the aim of this paper to discuss possible influences and describe the methods according the basic vibration theory which may help to reduce them. In all applications one must distinguish between the force indicated by the force measuring device and the force acting on the force transducer [3]. Therefore the mechanical structure of the arrangement used in dynamic application and the time dependence must be analysed first.

## 2 MECHANICAL STRUCTURE OF THE FORCE TRANSDUCER

The structure and signal flow of electrical force measuring devices are usually represented as a block diagram according to Fig.1 [4]. The static and dynamic properties of such a force measuring device are influenced by all components shown in the block diagram. The dynamic behaviour of force transducers can often be described by the simple spring mass model represented in Fig.1, i.e. by the motion of two masses with a spring of zero mass and stiffness $k_f$, and with a zero mass damper of damping coefficient $b_f$ connected in parallel (Voigt model). The masses of the transducer, which are spatially distributed, are discrete in this model and divided into an internal upper mass $m_{ti}$ and an internal lower mass $m_{bi}$. Force introduction often does not take place directly but through additional external masses denoted here by $m_{ta}$ and $m_{ba}$. Moreover, if it is assumed that the external masses are rigidly connected to the force transducer, the oscillation behaviour of the force transducer can be described by the movements of the upper and lower masses, $m_t$ and $m_b$. The displacements of the upper and lower masses from their rest positions are denoted by $x_t$ and $x_b$. $F_t$ and $F_b$ are the external forces acting on the upper and lower mass respectively.

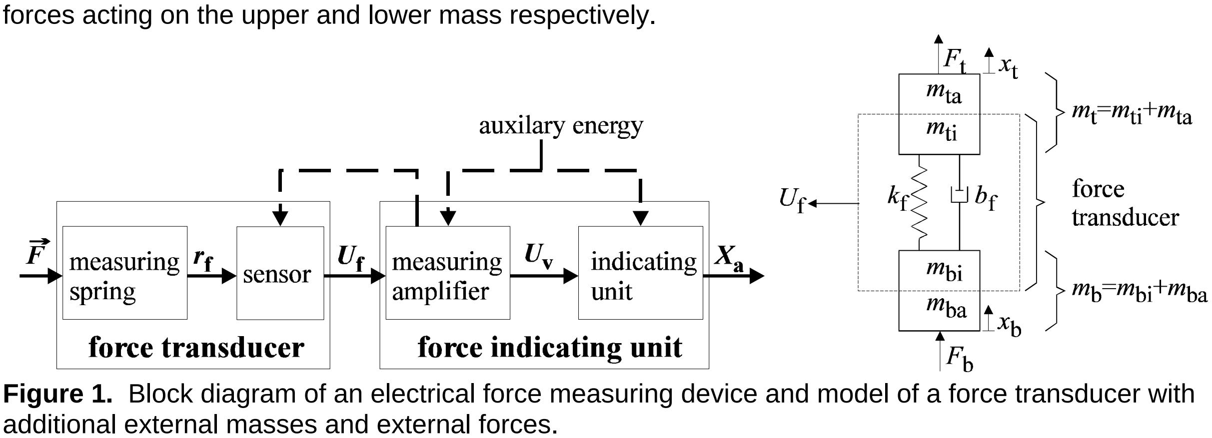


**Figure 1.** Block diagram of an electrical force measuring device and model of a force transducer with additional external masses and external forces.

The spring-mass-damper system is completely described by the following system of second order differential equations:

$$m_t \cdot \ddot{x}_t = F_t - k_f \cdot (x_t - x_b) - b_f \cdot (\dot{x}_t - \dot{x}_b) \quad (1a)$$
$$m_b \cdot \ddot{x}_b = F_b + k_f \cdot (x_t - x_b) + b_f \cdot (\dot{x}_t - \dot{x}_b) \quad (1b)$$

On the assumption that the sensor can be described as a non-delayed proportional element, the transducer output signal $U_f$ is directly proportional to the relative displacement $r_f = x_t - x_b$ .
The system of differential equations can, therefore, be rewritten as follows:

$$m_t \cdot \ddot{r}_f + b_f \cdot \dot{r}_f + k_f \cdot r_f = F_t - m_t \cdot \ddot{x}_b \quad (2a)$$
$$m_b \cdot \ddot{r}_f + b_f \cdot \dot{r}_f + k_f \cdot r_f = -F_b + m_b \cdot \ddot{x}_t \ . \quad (2b)$$

The static sensitivity of a force transducer $S_{f0}$ is defined as the quotient of the change in output signal $\Delta U_f$ and the change in active force $\Delta F_t$ it produces:
$$S_{f0} = \frac{\Delta U_f}{\Delta F_t} . \quad (3)$$

For the measurement of static force there are no time-dependent terms and the external force and spring force are in equilibrium. Assuming that the static sensitivity is constant and that there is a linear relationship between the displacement $r_f$ and output signal $U_f$ of the force transducer, with $\Delta U_f = U_f$ and $\Delta F_t = F_t = k_f \cdot r_f$ , it follows that:
$$k_f \cdot r_f = S_{f0}^{-1} \cdot U_f \ . \quad (4)$$
The derivation of Eq. 4 is valid subject to the condition that the force transducer is unloaded ($F_t$=0), not displaced ($r_f$=0) and that its output signal is $U_f$=0.

To avoid systematic errors, inertial forces and damping forces must be taken into consideration in dynamic force measurements. According to the above equations, this requires that the masses acting between the measuring spring and the points of force application be known, as well as the stiffness and damping of the force transducer. Since in many cases only the force introduced from above into the force transducer is of interest, it is sufficient to look at the first equation (2a) of the system of differential equations. Prerequisite for small uncertainties in the measurement of dynamic forces is that the force transducer can actually be described by the above model. Hence new methods and facilities are being developed for the dynamic calibration and for investigations of force measuring devices[1,2].

From Eq. 2a follows the interpretation that the force transducer is a 2$^{nd}$ order measuring component (PT$_2$-term) which has become important in numerous applications and will therefore be described in greater detail in the following sections. The advantage of this interpretation is that in practical applications the transient and resonance behaviour of the transducer can be estimated quite easily. If the base acceleration is negligible (e.g. for a rigid support), the results may also be used to compensate systematic deviations. But it must be pointed out that for larger base accelerations these must be measured too, and be accounted for in the compensation [5].

Another means of taking account of systematic deviations starts straight from the differential equation 1a. If the relationship between deformation and force transducer signal is given by Eq. 4, the equation may be rewritten as:

$$F_t = S_{f0}^{-1} \cdot U_f + m_t \cdot \ddot{x}_t + \frac{b_f}{k_f} \cdot S_{f0}^{-1} \cdot \dot{U}_f \quad (5)$$

According to this equation the dynamic force $F_t$ to be measured can be determined from the transducer signal $U_f$ and from the measurement of the acceleration $\ddot{x}_t$ of the effective mass $m_t$.

## 3 DYNAMIC RESPONSE OF THE FORCE TRANSDUCER

Besides the analysis of the mechanical structure of the whole mechanical setup, the time dependence of the force excitation in particular must be studied. Whilst sine-shaped force curves generally occur in fatigue tests, crash tests in the motor-car industry and pulse excitation in modal analysis suggest a pulsed force curve.

### 3.1 Transient behaviour for force impulse excitation

The following section considers the properties of a force transducer in order to describe its behaviour during a force impulse. Starting from Eq. 2a the free movement of a force transducer, i.e. no external forces and no base acceleration, is described by a homogeneous differential equation:

$$\ddot{r}_f + \frac{b_f}{m_t} \cdot \dot{r}_f + \frac{k_f}{m_t} \cdot r_f = 0 \quad \text{or} \quad \ddot{r}_f + 2d \cdot \dot{r}_f + w_0^2 \cdot r_f = 0 \quad (6)$$

For reasons of simplicity, the usual denotations of oscillator theory have been introduced:

damping coefficient $d = \frac{b_\mathrm{f}}{2m_\mathrm{t}}$, characteristic angular frequency $w_0 = \sqrt{\frac{k_\mathrm{f}}{m_\mathrm{t}}}$ and characteristic frequency $f_0 = \frac{w_0}{2p}$.

In the case of force transducers very weak damping generally prevails, i.e. $b_\mathrm{f} << 2 \cdot \sqrt{m_\mathrm{t} \cdot k_\mathrm{f}}$ is valid. The solution of the homogeneous differential equation can then be stated as follows:

$$r_\mathrm{f}(t) = r_0 \cdot e^{-Dw_0 t} \cdot \cos(w_d t - j) \text{ with } w_d = \sqrt{w_0^2 - d^2} = w_0\sqrt{1 - D^2} \quad (7)$$

Here $w_d$ is the characteristic angular frequency of the damped oscillator in $s^{-1}$, $f_d = \frac{w_d}{2p}$ is the characteristic frequency of the damped oscillator in $s^{-1}$ and $D = \frac{d}{w_0}$ the damping factor. The constants $r_0$ and $j$ are determined from the initial conditions. Equation (7) describes the abrupt unloading of a force transducer as an exponentially decaying oscillatory process, which is plotted in Fig. 2a normalised to the initial value $r_0$.

For abrupt loading of a force transducer with an initial force $F_\mathrm{t}(t)=F_0$, the differential equation in the case of an unaccelerated base is:

$$\frac{1}{w_0^2} \cdot \ddot{r}_\mathrm{f} + \frac{2D}{w_0} \cdot \dot{r}_\mathrm{f} + r_\mathrm{f} = \frac{F_0}{k_\mathrm{f}} (= r_0) \quad . \quad (8)$$

Here $r_0$ is the increase in force normalised to the transducer stiffness. For the transducer in the present case of a periodically oscillating adjustment (D<1), the normalised response is [6]:

$$\frac{r_\mathrm{f}}{r_0} = 1 - \frac{w_0}{w_d} e^{-Dw_0 t} \cos(w_d t - j) \text{ with } w_d = w_0\sqrt{1 - D^2} \text{ and } j = \arctan\left(\frac{D}{\sqrt{1 - D^2}}\right) \quad (9)$$

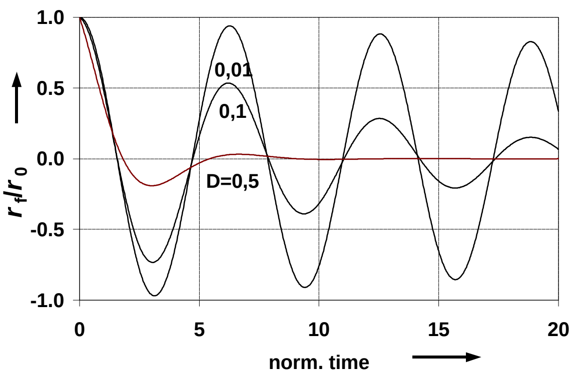


**Figure 2. a)** Normalised response $r_\mathrm{f}(t)/r_0$ according to Eq.7 as a function of the normalised time $tw_0$ for various damping factors $D$<1.

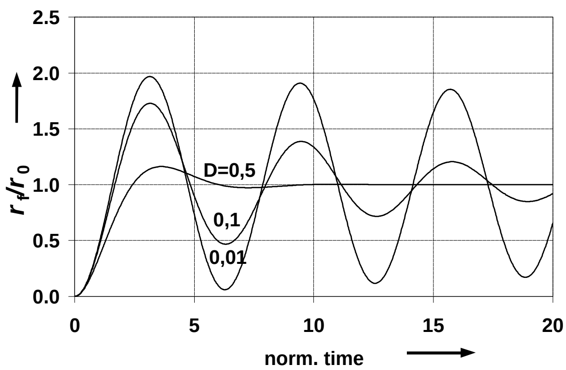


**Figure 2. b)** Normalised response $r_\mathrm{f}(t)/r_0$ according to Eq.9 as a function of the normalised time $tw_0$ for various damping factors $D$<1.

The normalised response of a force transducer is shown in Fig.2b for various damping factors. It may be clearly seen that for transducers with small damping factors, e.g. $D$=0.01, prolonged periods of oscillation occur.

## 3.2 Frequency response of a force transducer

During the measurement of dynamic forces the transducer is subjected to external force excitation. In the case of the unaccelerated lower mass, from Eq.2a we may derive the equation:

$$m_\mathrm{t} \cdot \ddot{r}_\mathrm{f} + b_\mathrm{f} \cdot \dot{r}_\mathrm{f} + k_\mathrm{f} \cdot r_\mathrm{f} = F_\mathrm{t}(t) = \hat{F} \cdot \cos(w \cdot t + j_\mathrm{F}) = \mathrm{Re}\{\underline{\hat{F}} \cdot e^{jw \cdot t}\} \text{ with } \underline{\hat{F}} = \hat{F} \cdot e^{jj_\mathrm{F}} \quad (10)$$

Whilst the solution of the homogeneous equation characterises the transient behaviour, the continuous state is described by the particular solution. Since any exciting force can be composed of sine-shaped forces according to the superposition principle, it is sufficient to consider sine-shaped forces here. Starting from the particular solution:

$$r_{\mathrm{f}}(t) = \hat{r} \cdot \cos(w \cdot t + j_{\mathrm{r}}) = \mathrm{Re}\left\{ \underline{\hat{r}} \cdot e^{jw \cdot t} \right\} \text{ with } \underline{\hat{r}} = \hat{r} \cdot e^{jj_{\mathrm{r}}} \qquad (11)$$

and substituting in the differential equation, the complex frequency response follows:

$$\frac{k_{\mathrm{f}} \cdot \underline{\hat{r}}}{\underline{\hat{F}}} = \frac{1}{1 - \frac{m_{\mathrm{t}}}{k_{\mathrm{f}}} \cdot w^2 + j \frac{b_{\mathrm{f}}}{k_{\mathrm{f}}} \cdot w} = \frac{1}{1 - \left( \frac{w}{w_0} \right)^2 + j2D \frac{w}{w_0}} \qquad (12)$$

or for the amplitude and phase shift [6]:

$$\frac{k_{\mathrm{f}} \cdot \hat{r}}{\hat{F}} = \frac{1}{\sqrt{\left( 1 - \left( \frac{w}{w_0} \right)^2 \right)^2 + 4D^2 \left( \frac{w}{w_0} \right)^2}} \text{ and } j_{\mathrm{rF}} = j_{\mathrm{r}} - j_{\mathrm{F}} = -\arctan\left( \frac{2D \frac{w}{w_0}}{1 - \left( \frac{w}{w_0} \right)^2} \right) \qquad (13a,b)$$

The dependence of the amplification function $V\left( \frac{w}{w_0}, D \right) = \frac{k_{\mathrm{f}} \cdot \hat{r}}{\hat{F}}$ on the frequency relation $\frac{w}{w_0}$ is shown for various damping factors *D* in Fig.3a. Figure 3b shows the matching phase curve. Force transducers usually have a very small damping factor of 0<D<0,01. The amplification function shows that the resonance behaviour of a transducer can lead to large systematic deviations. The relationship between the resonance frequency $f_r$ and characteristic frequency $f_0$ of a force transducer is given by:

$$f_r = f_0 \sqrt{1 - 2D^2} \qquad (14a)$$

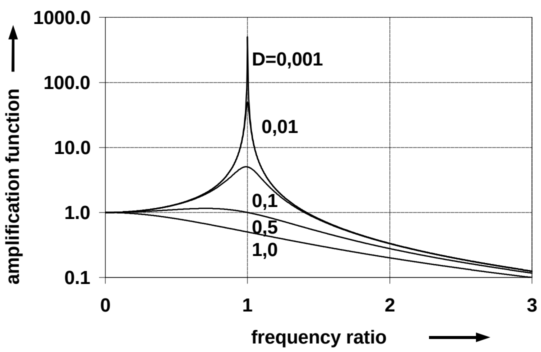


**Figure 3. a)** Amplification function (amplitude response) V(*w/w*$_0$,*D*) according to Eq.13a for various damping factors *D* as a function of the frequency ratio *w/w*$_0$.

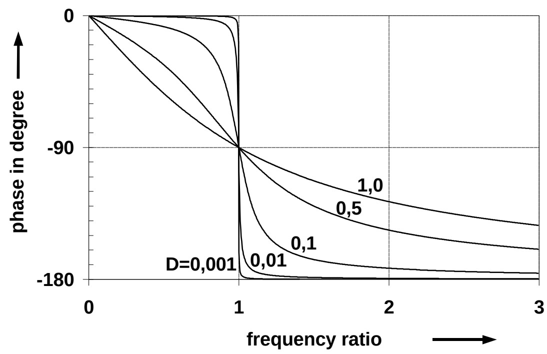


**Figure 3. b)** Phase shift $\varphi_{rF}$ according to Eq.13b for various damping factors *D* as a function of the frequency ratio *w/w*$_0$.

For the unloaded force transducer the resonance frequency $f_r$ lies in the kHz range and when coupled with external masses may show values below 100 Hz. It is therefore possible that even at low frequencies systematic deviations of several percent may occur. When a transducer has negligible damping, its resonance frequency $f_r$ and eigenfrequency $f_d$ are equal to the characteristic frequency $f_0$ and change with the coupled mass:

$$f_r = f_{\mathrm{d}} = f_0 = \frac{1}{2p} \cdot \sqrt{\frac{k_{\mathrm{f}}}{m_{\mathrm{ti}} + m_{\mathrm{ta}}}} \quad . \qquad (14b)$$

Here $m_{ti}$ is the inner (co-oscillating) mass of the transducer and $m_{ta}$ the external additional mass. The fundamental eigenfrequency $f_{0g}$ of a force transducer is defined as the eigenfrequency without external additional mass:

$$f_{0g} = \frac{1}{2p} \cdot \sqrt{\frac{k_f}{m_{ti}}} \quad . \qquad (14c)$$

## 4 USE OF DYNAMICALLY CALIBRATED FORCE MEASURING DEVICES

A condition for small uncertainties in the measurement of dynamic forces is that the force transducer can be described by the above model. In particular, it is important to find out whether the static sensitivity and the sensitivity determined dynamically are identical, and whether the dynamic behaviour of a transducer may be described by the model shown in Fig.1. These conditions are however only approximately satisfied since the mechanical deformation body of the transducer represents a continuum with uniformly distributed mass, spring and damping elements. Moreover, there may also be deviations between the displacement measured by the measuring sensor and the real displacement. On the one hand this is because the measuring sensors integrate over a given area of the mechanical deformation body, and on the other because of the cut-off frequency of the sensor. Thus the cut-off frequencies of strain gauge devices lie in the region of some 10 kHz to ca. 200 kHz and depend on their length and the adhesive layer [7].

Furthermore, the model completely ignores the fact that very many of the measuring springs employed in practice do not describe a strictly linear displacement (bending springs etc.), and they may also have eccentric centres of gravity $m_t$ and $m_b$. Where the transducer characteristics are nonlinear, the sensitivity of the force transducer depends on the force. This is accounted for by least square calculations with up to third degree polynomial functions. For dynamic measurements force transducers should have linear characteristics where possible [3].

For this purpose a measuring facility has been designed to generate well-defined time-dependent forces which can be accurately calculated [1,2]. The force transducer to be calibrated is mounted on a shaker and a load mass is screwed on to the transducer. The dynamic force can then be determined from the acceleration of the effective dynamic mass which consists of the load mass $m_l$ and the so-called end mass $m_e$ of the force transducer. The end mass $m_e$ is the internal upper mass $m_{ti}$ and the load mass $m_l$ is the additional external mass $m_{ta}$ in the model (Fig.1). The dynamic sensitivity of the force measuring device is the ratio of the output signal of the device divided by the dynamic force acting on the sensing element of the force transducer. The methods and results of the dynamic calibration of force measuring devices have been described in previous contributions in more detail. In dynamic applications, the results can be used to compensate and estimate the errors of dynamic force measurement. A distinction must be made between the dynamic properties of the force sensor, the measuring amplifier, the mechanical coupling of the force sensor and electronic devices such as the FFT analyser or AD converter cards used for signal analysis.

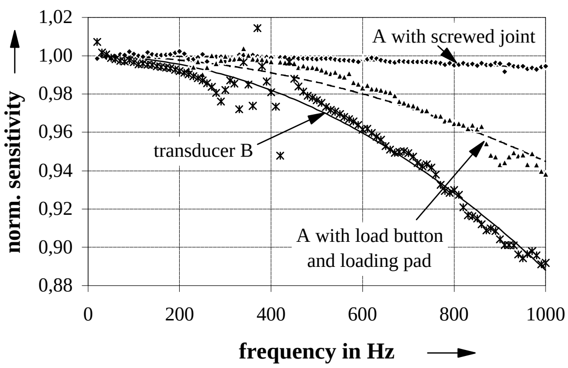


**Figure 4. a)** Normalised dynamic sensitivity of a double-beam type transducer B and of a shear type transducer A with different kinds of force introduction.

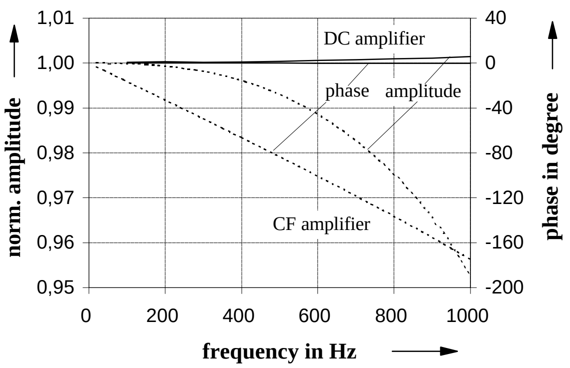


**Figure 4. b)** Amplitude and phase response of a DC and a carrier frequency amplifier.

The main mechanical influences on the dynamic properties of force measuring devices are those from the mechanical structure of the force transducer and from force introduction. As shown in Fig. 4a, the two transducers A and B, both of high static quality and with the same static capacity of 2 kN, may exhibit completely different dynamic behaviour. These differences can be explained by the mechanical structure of the transducer. Furthermore, transducer B shows more irregularities than transducer A, because transducer B is more sensitive to side forces. Figure 4a also shows that force introduction can lead to a significant change in sensitivity. If the force is introduced by a load button with loading pad, the stiffness of the force introduction is reduced compared with a screwed joint [1,4].

DC and carrier frequency amplifiers are used to measure the detuning of a Wheatstone measuring bridge. In dynamic applications, the resistances of strain gauge force transducers are amplitude modulated. For dynamic calibration of the amplifiers it is necessary to simulate well-defined bridge detuninig. With special electronic bridges it is possible to investigate the dynamic properties of amplifiers used in strain gauge force transducers [1]. Figure 4b shows the amplitude and phase response of a DC and a 5 kHz carrier frequency amplifier. In dynamic applications the frequency response of the amplifier can be directly taken into account.

In many dynamic applications piezoelectric force measuring devices are used. The investigations have shown that the dynamic sensitivity of the piezoelectric force measuring devices is mainly influenced by the frequency response of the charge amplifier [1].

The frequency response of the signal analyser can be determined with AC voltage standards, or AC-calibrated voltmeters, and can be taken into account in the measurements [1]. The frequency response of the measuring amplifier can also be allowed for when dynamic force measurement is applied in practice. The dynamic sensitivity of the force sensor, which depends on the internal structure of the sensor and of the force sensor coupling, can change if the situation in practical application differs from that prevailing upon calibration. It is, therefore, necessary that the same coupling be used, both for the determination of the dynamic sensitivity of the force sensor and in practical application. The resonance behaviour, which can also be determined by means of the calibration facilities, can be used to estimate the errors in practical applications.

## 5 CONCLUSION

This paper demonstrates how dynamically calibrated force transducers can be used in dynamic applications and which methods can be applied to reduce the measurement uncertainty. Different influences are described and analysed with respect to applications in the field of dynamic force measurement.

It is shown that in many cases the force indicated by the force transducer can noticeably differ from the force which must be determined in dynamic application. The deviation increases with increasing frequency and mainly depends on the stiffness and the mass distribution in this particular case. A reduction of these systematic influences can be achieved by application of a theoretical model which describes the vibration behaviour of the system. Methods are discussed which allow the systematic influences to be reduced. If the forces are measured only with force transducers, the resonance behaviour must be well known, and it must be reproducible so that the systematic influences can be taken into account. Moreover, it must be borne in mind that, in addition to the resonance behaviour of the force transducer, the resonance behaviour of the surrounding mechanical structure can significantly influence the measurement results. Another method takes the acceleration of the acting masses into account. This demonstrates that dynamic force measurement is strongly related to acceleration measurement.

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