

CHARACTERIZATION OF THE MAGNETIC FIELD OF A WEIGHING INSTRUMENT

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Abstract: A mathematical model of the magnetic interaction between a weighing instrument and a weight and measurements of the distribution of the magnetic field in the space above a top loading weighing instrument are presented. Four parameters have been evaluated that characterize sufficiently well the magnetic property of the instrument, provided the weight's magnetic susceptibility and magnetization are homogeneously distributed over its volume. The magnetic field of the weighing instrument is modelled by a magnetic dipole field. The integration over the weight's volume was made by stepwise numerical subdivision of the volume into subvolumes.

Keywords: magnetic field, weighing instrument.

1 INTRODUCTION

The calibration of a mass standard by the use of a weighing instrument is based on the gravitational force on the mass standard. The presence of other forces, for example magnetic forces, will lead to undesired deviations of the measurement result. For this reason, new requirements for weights concerning their magnetic susceptibility and magnetization will be laid down in a forthcoming OIML recommendation [1]. Measurement methods and instruments for characterizing these magnetic properties of weights have been developed [2] and will be described in the recommendation.

The magnetic properties of weights however are only a source of a force, if an inhomogeneous ambient magnetic field or other magnetic materials are surrounding the weight. Measurements of the magnetic field above the pan of a weighing instrument and rough estimates of the forces on a weight of homogeneous magnetic susceptibility have previously been reported [3].

Here, a method for modelling the magnetic field and field gradients on a weighing instrument, based on a few number of measurements, as well as the evaluation of the change of apparent mass due to magnetic forces will be presented.

2 MATHEMATICAL MODEL

The magnetic field of a weighing instrument has been modelled in a first approach by that of a magnetic dipole, located somewhere below the balance pan, see Figure 1. The field of the dipole, in this case, has to be modelled in a rigorous way, that means that the length of the dipole cannot be assumed to be negligibly small compared to the distance of the field point, and the orientation of the dipole in respect to the balance pan and to the weight, the azimuth and polar angles, have to be taken into account.

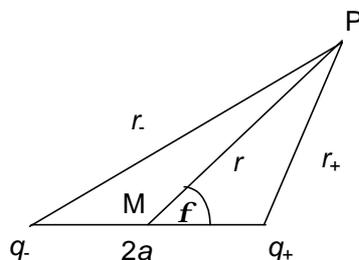


Figure 1. Magnetic dipole in analogy to an electric dipole with q_+ and q_- the magnetic charges (north and south, respectively), $2a$ their distance, M point of the center of the dipole, P the field point with its distance r from M , r_+ from the q_+ charge and r_- from the q_- charge, f angle between field point direction and dipole.

In a cartesian laboratory coordinate system (z-axis vertical) the z-component of the magnetic field H_z of a dipole is:

$$H_z = -\frac{m_m^*}{4\mathbf{p}} \frac{1}{2a} \left((z_{q+} - z_m) \left(\frac{1}{r_+^3} + \frac{1}{r_-^3} \right) - (z_p - z_m) \left(\frac{1}{r_+^3} - \frac{1}{r_-^3} \right) \right) \quad (1)$$

m_m^* magnetic dipole moment, r_+ and r_- distances between the field point and the north- and south-pole, respectively, z_{q+} and z_p the z-coordinates of the north pole of the dipole and of the field point, respectively, z_m the z-coordinate of the center of the dipole.

The formulas for H_x and H_y look similar to that of H_z - the coordinates z_{q+} and z_p are replaced by the corresponding x- and y- coordinates. For further considerations, we define the orientation of the dipole in respect to the cartesian coordinates of the field point by introducing the dipole's polar angle \mathbf{J} against the z-axis and its azimuth angle \mathbf{y} in the x-y-plane. We have then:

$$x_{q+} - x_m = a \sin \mathbf{J} \cos \mathbf{y} \quad y_{q+} - y_m = a \sin \mathbf{J} \sin \mathbf{y} \quad z_{q+} - z_m = a \cos \mathbf{J}$$

r_+ and r_- are given by:

$$r_+ = r \sqrt{1 + \frac{a^2 - 2ar \cos \mathbf{j}}{r^2}} \quad r_- = r \sqrt{1 + \frac{a^2 + 2ar \cos \mathbf{j}}{r^2}}$$

$$\text{with: } r = \sqrt{(x_p - x_m)^2 + (y_p - y_m)^2 + (z_p - z_m)^2}$$

$$\text{and: } \cos \mathbf{j} = \cos \mathbf{a}_1 \cos \mathbf{a}_2 + \cos \mathbf{b}_1 \cos \mathbf{b}_2 + \cos \mathbf{g}_1 \cos \mathbf{g}_2$$

$$\cos \mathbf{a}_1 = \sin \mathbf{J} \cos \mathbf{y} \quad \cos \mathbf{a}_2 = \frac{x_p - x_m}{r}$$

$$\cos \mathbf{b}_1 = \cos \mathbf{J} \sin \mathbf{y} \quad \cos \mathbf{b}_2 = \frac{y_p - y_m}{r}$$

$$\cos \mathbf{g}_1 = \cos \mathbf{J} \quad \cos \mathbf{g}_2 = \frac{z_p - z_m}{r}$$

The z-component of the gradient of H_z is:

$$\frac{\partial H_z}{\partial z} = -\frac{m_m^*}{4\mathbf{p}} \frac{1}{2a} \left(-6a^2 \cos^2 \mathbf{J} \left(\frac{1}{r_+^5} + \frac{1}{r_-^5} \right) (z_p - z_m) + 3 \left(\frac{1}{r_+^5} - \frac{1}{r_-^5} \right) \left(a^2 \cos^2 \mathbf{J} + (z_p - z_m)^2 \right) - \left(\frac{1}{r_+^3} - \frac{1}{r_-^3} \right) \right) \quad (2)$$

The z-component of a force on a sample with a magnetic susceptibility c and a magnetization M^* in a magnetic field with non-vanishing gradient in vacuum is:

$$F_z = -\frac{\mathbf{m}_0}{2} \int \frac{\partial}{\partial z} (\mathbf{cH} \cdot \mathbf{H}) dV - \mathbf{m}_0 \int \frac{\partial}{\partial z} (\mathbf{M}^* \cdot \mathbf{H}) dV \quad (3)$$

μ_0 vacuum permeability, V volume, \mathbf{H} vector of the local magnetic field, \mathbf{M}^* vector of the local permanent magnetization of the weight.

If we consider a volume ΔV being small compared to the change of the field in z-direction, if c and M_z^* of the weight are homogeneously distributed and if we take into account the magnetic susceptibility of air c_a , the magnetic field of the balance \mathbf{H}_b and a (constant) ambient magnetic field, \mathbf{H}_0 , we have approximately:

$$F_z = -\mathbf{m}_0 \Delta V (\mathbf{c} - \mathbf{c}_a) \left| \mathbf{H}_b + \mathbf{H}_0 \right| \frac{\partial |\mathbf{H}_b|}{\partial z} - \mathbf{m}_0 \Delta V M_z^* \frac{\partial H_{bz}}{\partial z} \quad (4)$$

The force on a sample can numerically be calculated by applying equation (4) to an appropriate number of subvolumes of the sample and taking the sum of the corresponding differential forces.

3 EXPERIMENTAL CONDITIONS

The weighing instrument is a top loading 10 kg balance with a resolution of 1 mg. The magnetic field is measured with a fluxgate magnetometer with ranges from 0,03 μT to 100 μT and an accuracy of 2,5% of full scale. A device has been manufactured that allows to displace the fluxgate probe in a cartesian coordinate system and measure the coordinates on scales with millimeter divisions.

4 MAGNETIC FIELD DISTRIBUTIONS OF A WEIGHING INSTRUMENT

The cartesian magnetic field components H_x , H_y and H_z have been measured (in terms of the magnetic induction $B=\mu_0H$) at three different z-levels and at five points in each x-y-plane above the pan of the weighing instrument, 45 measurements in total. The size of the pan is 13 cm x 13 cm, the measured area 5 cm x 5 cm and the z-levels are 1,8 cm, 3,6 cm and 7,6 cm.

The values of the dipole moment and the six space-parameters of the dipole: the coordinates of its center x_m , y_m and z_m , its length $2a$, the azimuth angle γ and the polar angle θ have been adjusted to the measured data of B_x , B_y and B_z at its respective coordinates.

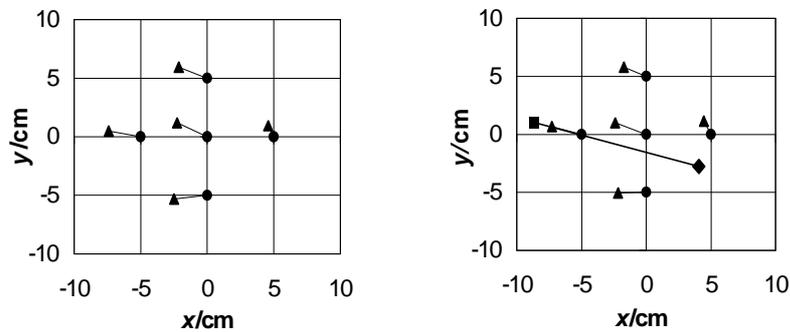


Figure 2. Measured (left) and calculated (right) magnetic fields in the x-y-plane (at $z=1,8$ cm) as vector lines drawn from the x-y-coordinates (dots) of the measurement points to points (triangles) giving the values of the magnitude and direction of the field. The largest absolute B_x value is 50 μT , the largest absolute B_y value is 24 μT . The north and south points of the dipole (right) are shown as square and rhomb and they are connected by a line.

Figure 2 shows the measured and calculated magnetic field vectors in the x-y-plane and Figure 3 shows data in the x-z-plane. The data are given without the earth magnetic field components. The coordinate origin of the x- and y-axes is the center of the pan, the origin of the z-axis is the pan surface. The deviations of the adjusted data from the measured ones give a standard deviation of 3 μT . This good agreement shows that a dipole model approximates reasonably well the magnetic field of the weighing instrument used here.

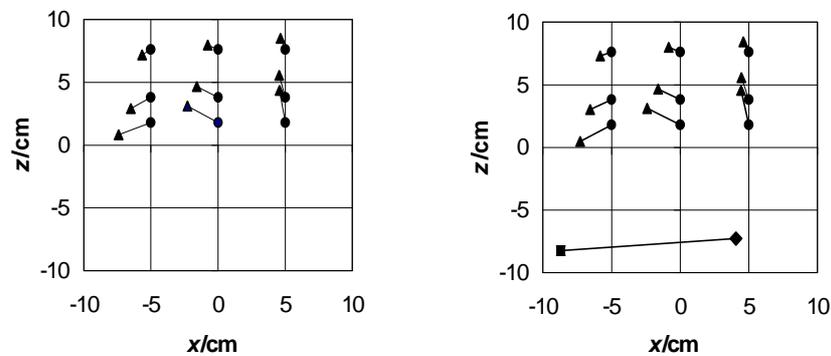


Figure 3. Measured (left) and calculated (right) magnetic fields in the x-z-plane (at $y=0$). For further information see Fig.2. The largest absolute B_z value is 51 μT , the largest absolute B_x value is 50 μT .

5 PARAMETERS OF THE MAGNETIC FIELD

If we wish to estimate the magnetic force between a weight and a weighing instrument and the corresponding change of apparent mass, it could be useful to attribute parameters to the weighing instrument in a similar way as weights are characterized by their magnetic susceptibility c and their magnetization M^* . The weighing instrument generates a magnetic field with an inhomogeneous distribution in space. If the magnetic field above the instrument's pan is measured and modelled in a similar way as shown above, four volumic parameters may be calculated that allow to estimate the apparent mass deviations in a simple way. A condition is, that the magnetic susceptibility and the magnetization of the weight can be assumed to be homogeneous over its volume. The fields are written in terms of magnetic induction (unit μT). We write equation (4) in the following form:

$$\Delta m_H = -\frac{1}{g\mathbf{m}_0} \left[c' \sum_V |\mathbf{B}_b + \mathbf{B}_0| \frac{\partial |\mathbf{B}_b|}{\partial z} \Delta V + M_z * \sum_V \frac{\partial B_{bz}}{\partial z} \Delta V \right] \quad (5)$$

with $c' = c - c_a$. We define the parameters:

$$K_1(V) = -\frac{1}{g\mathbf{m}_0} \sum_V \left(B_{bx} \frac{\partial B_{bx}}{\partial z} + B_{by} \frac{\partial B_{by}}{\partial z} + B_{bz} \frac{\partial B_{bz}}{\partial z} \right) \Delta V \quad (6)$$

$$K_x(V) = -\frac{1}{g\mathbf{m}_0} \sum_V \frac{\partial B_{bx}}{\partial z} \Delta V \quad (7)$$

$$K_y(V) = -\frac{1}{g\mathbf{m}_0} \sum_V \frac{\partial B_{by}}{\partial z} \Delta V \quad (8)$$

$$K_z(V) = -\frac{1}{g\mathbf{m}_0} \sum_V \frac{\partial B_{bz}}{\partial z} \Delta V \quad (9)$$

The numerical summation is performed over the volume above the instrument's pan, the volume that is kept by the weight during the weighing process. The apparent mass deviation results from the following equation:

$$\Delta m_H(V) = c' (K_1(V) + K_x(V)B_{0x} + K_y(V)B_{0y} + K_z(V)B_{0z}) + M_z * K_z(V) \quad (10)$$

To give some examples, Table 1 shows the parameters of the weighing instrument used in Chapter 4 for several cylindrical weights with a height-to-diameter ratio of 1,44. The numerical summation is made by subdividing the cylinder's volume into subvolumes shaped as hexagonal columns with heights decreasing from top to bottom. Figure 4 shows an example of the subdivision into 19 hexagons. The total surface area of the base is the same as that of the cylinder. The number of subvolumes was increased stepwise, until the results converged.

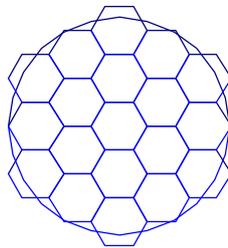


Figure 4. Example for the subdivision of a circular cross area into 19 hexagons of the same surface area.

A cylindrical 1 kg weight, for example, with $c = 0,01$ and $M_z^* = + 3 \mu\text{T}$ centred on the pan of the instrument and at an ambient earth field of $B_{0x} = -11,5 \mu\text{T}$; $B_{0y} = -4,5 \mu\text{T}$ and $B_{0z} = -34,0 \mu\text{T}$, has a deviation of 0,017 mg. For the chosen parameters of the weight and the instrument used here, the

contribution of the magnetization of the weight is the largest contribution to the deviation of apparent mass. If we take a 10 kg weight made of grey cast iron with typical parameters $c = 4$ and $\mu_0 M = + 50 \mu\text{T}$, we obtain a deviation of 8,9 mg. Here, the contribution of the susceptibility is the largest one. The contribution of the field of the weighing instrument however has a sign opposite to the one of the earth magnetic field. They should have the same sign, if the polarity of the instrument's dipole is inverted. In this case, the change of apparent mass is 11,8 mg.

Table 1. Parameters K_x , K_y , K_z and K_1 of the mentioned weighing instrument for different cylindrical weights with a height-to-diameter ratio of 1,44.

m/kg	V/cm^3	$K_x/\text{mg}\mu\text{T}^{-1}$	$K_y/\text{mg}\mu\text{T}^{-1}$	$K_z/\text{mg}\mu\text{T}^{-1}$	K_1/mg
0,001	0,125	$-1,54 \times 10^{-05}$	$7,54 \times 10^{-06}$	$8,25 \times 10^{-06}$	$1,55 \times 10^{-03}$
0,01	1,25	$-1,39 \times 10^{-04}$	$6,67 \times 10^{-05}$	$7,60 \times 10^{-05}$	$1,29 \times 10^{-02}$
0,1	12,5	$-1,12 \times 10^{-03}$	$5,22 \times 10^{-04}$	$6,32 \times 10^{-04}$	$9,14 \times 10^{-02}$
1	125	$-7,42 \times 10^{-03}$	$3,28 \times 10^{-03}$	$4,29 \times 10^{-03}$	$5,18 \times 10^{-01}$
10	1250	$-3,53 \times 10^{-02}$	$1,43 \times 10^{-02}$	$2,05 \times 10^{-02}$	$2,34 \times 10^{00}$

For a rectangular weight, with a base area that is not quadratic, the parameters have to be evaluated for different angles by which the weight is turned around its vertical axis. For a rectangular 10 kg weight made of grey cast iron with the ratio $a:b = 2:1$ of the dimensions of its base and with the weighing instrument described above, the change of apparent mass was found to be a factor of 3 different between two positions turned by an angle of 90° .

It has to be mentioned, that the parameters K_x , K_y , K_z and K_1 of the weighing instrument depend in general on the size, the volume and on the position of the weight. These parameters increase less than proportionally toward smaller volumes of similarly sized weights with a height-to-diameter ratio larger than one, because the magnetic field of the weighing instrument decreases with the distance from its pan. If the weight is moved in the horizontal plane, the parameters may change considerably depending on the horizontal field distribution, and for rectangular weights they may differ depending on the angle by which the weight is turned around its vertical axis. In cases, where the magnetic susceptibility or the magnetization of the weight is strongly inhomogeneous or anisotropic, the magnetic interaction between weight and weighing instrument cannot be characterized by the parameters given above. In this case, models of the distributions of the magnetic susceptibility and magnetization of the weight may be developed, and a numerical integration according to equation (3) has to be carried out.

6 CONCLUSION

It has been shown, that the magnetic field distribution in the space above a weighing instrument can be modelled with reasonable mathematical and computational effort. The expected deviations of the apparent mass of a weight with homogeneously distributed magnetic susceptibility and magnetization can be estimated by use of four parameters developed from the distribution of the magnetic field of the weighing instrument.

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