

## HARMONIC DISTORTION AND ADC

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*Abstract: The influence of different ADC nonlinearities on the measurement of ADC intermodulation products is discussed. It is shown that the amplitudes of harmonics and intermodulation products can be predicted only if the low order nonlinearity of the transfer curve is dominant and if a sufficient level of input noise or dither is used.*

*Keywords: ADC measurement, harmonics, spurious, dither*

### 1 INTRODUCTION

The ADC nonlinearities produce persistent sine waves error signals that affect more severely ADC performance than noise. The influence of some nonlinearities, such as quantization and DNL can be minimized by using dither or input noise that spread the spectrum of the corresponding error signals and so transfer the persistent sine waves power to noise. However, the influence of others, such as those of the low order nonlinearity of the transfer curve or of the dynamic behavior of the S/H circuit remains. The sine waves error signals can be divided into two groups: mixing products of the input signal (harmonics and intermodulation products) and mixing products between the input and the sampling frequency (spurious). The harmonics and intermodulation products could be considered as low order mixing products of the input produced by the nonlinear smoothed transfer curve. Being so not only the frequency, but also the amplitude could be predicted. The mth-order mixing product would change according to the mth-order law about its frequency and amplitude. If this is valid, the ADC's low order non-linearity could be described as in the case of analog circuits by intercept points or by the polynomial expression that describe the smoothed transfer curve [1,2]. Other attempts were made to compute the INL from harmonics amplitude [3]. In this paper we present numerical simulation and experimental results corresponding to preliminary work in this field. Our aim is to present our experience and open the discussion about the proper definition of harmonics and spurious. The main question is: when the signal changes only its frequency according to the mth-order law and its amplitude is random, is it an harmonic or a spurious component?

### 2 THEORY

The smoothed ADC's transfer curve or the nonlinearity of an analog IC can be expressed by the low order polynomial expression

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n, \quad (1)$$

where y is the equivalent the output signal, x the input signal and n the order of the circuit nonlinearity. Such a circuit produces harmonics to the order n. If the input signal is given by

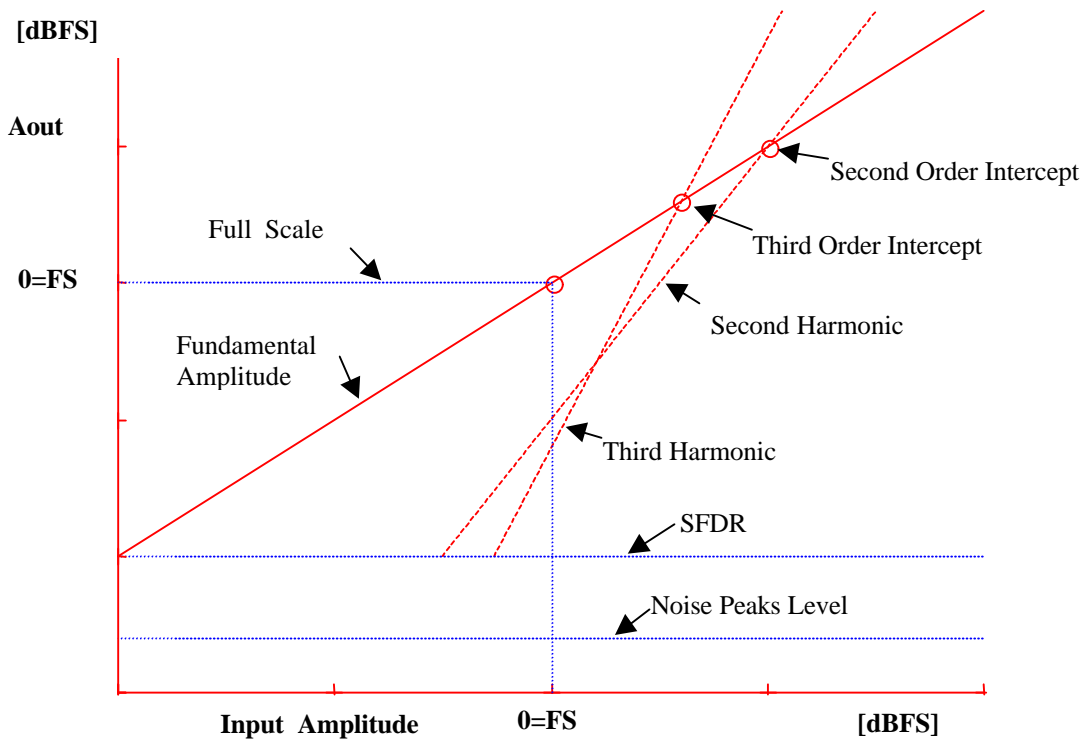
$$x = A \cos(\omega t), \quad (2)$$

the amplitudes of the harmonic terms computed from (1) are shown in Tab. 1. Even coefficients of the polynomial expression produce even harmonic terms odd coefficients produce odd harmonic terms. The n-th coefficient contributes to the n-th harmonic term and lower harmonic terms. These low order terms have bigger amplitudes than the n-th term, see as an example the influence of coefficient  $a_5$ . The corresponding amplitude for the 3<sup>rd</sup> harmonic term is about five times bigger than the amplitude of 5<sup>th</sup> harmonic term.

frequency	amplitude
2 $\omega$	$0.5 a_2 A^2 + 0.125 a_4 A^4 + 0.47 a_6 A^6 + 0.44 a_8 A^8$
3 $\omega$	$0.25 a_3 A^3 + 0.31 a_5 A^5 + 0.33 a_7 A^7 + 0.72 a_9 A^9$
4 $\omega$	$0.125 a_4 A^4 + 0.19 a_6 A^6 + 0.22 a_8 A^8$
5 $\omega$	$0.062 a_5 A^5 + 0.11 a_7 A^7 + 0.14 a_9 A^9$
6 $\omega$	$0.031 a_6 A^6 + 0.062 a_8 A^8$
7 $\omega$	$0.016 a_7 A^7 + 0.035 a_9 A^9$

Tab.1. Amplitudes of harmonics terms, for n=9 in (1) and an input signal given by (2).

In the case of analog circuits the order of a polynomial expression is mostly limited to 3 and the mth-order law is used to determine the output amplitudes and frequencies [2]. The output amplitudes depend on the amplitude of the input signal (m dB for the m-order product per dB of the fundamental amplitude), and output frequencies are related to the input signal frequency (the mf Hz shift for the m-order product results from a f Hz shift of the fundamental). The quality of an analog circuit is described in this case by the intercept point, which is the theoretical signal amplitude at which the fundamental and corresponding harmonic terms are equal, see Fig. 1. It would be nice if the same approach could be used for ADCs, and moreover, if in this case we could compute the coefficients of the transfer curve from the measured amplitudes of the harmonic terms.



**Fig. 1.** Harmonic distortion and intercept points. Prerequisite: polynomial coefficients of the 4<sup>th</sup> order and higher and the nonlinearity caused by saturation at full scale are neglected.

The same analysis can be done according to intermodulation distortions, where we suppose the input signal to be the sum of two sinusoidal signals, with amplitudes A, B and frequencies  $\omega_1, \omega_2$ :

$$x = A \cos(\omega_1 t) + B \cos(\omega_2 t). \tag{3}$$

The amplitudes of some intermodulation products are shown in Tab. 2. The resulting amplitudes depend on both amplitudes, A and B, even in the case of components such as  $2\omega_1, 3\omega_1, \dots, 2\omega_2, 3\omega_2 \dots$  that can be counted as harmonic products. In fact, when the order of the polynomial expression is equal or higher than 5 the amplitudes of components  $2\omega_1, 3\omega_1$  will change when B varies, even if the amplitude A remains constant.

frequency	amplitude
$2\omega_1$	$0.5 a_2 A^2 + 0.5 a_4 A^4 - 1.5 a_4 A^2 B^2 + 0.47 a_6 A^6 + 3.7 a_6 A^4 B^2 + 2.8 a_6 A^2 B^4$
$3\omega_1$	$0.75 a_3 A^3 + 0.31 a_5 A^5 + 3.75 a_5 A^3 B^2 + 0.33 a_7 A^7 + 3.3 a_7 A^5 B^2 + 3.3 a_7 A^3 B^4$
$\omega_1 + \omega_2$	$a_2 A B + 1.5 a_4 A^3 B + 1.5 a_4 A B^3 + 1.9 a_6 A^5 B + 5.6 a_6 A^3 B^3 + 1.9 a_6 A B^5$
$2\omega_1 + \omega_2$	$0.75 a_3 A^2 B + 1.25 a_5 A^4 B + 1.9 a_5 A^2 B^3 + 1.6 a_7 A^6 B + 6.6 a_7 A^4 B^3 + 3.3 a_7 A^2 B^5$
$\omega_1 + 2\omega_2$	$0.75 a_3 A B^2 + 1.25 a_5 A B^4 + 1.9 a_5 A^3 B^2 + 3.3 a_7 A^5 B^2 + 6.6 a_7 A^3 B^4 + 1.6 a_7 A B^6$

Tab.2. Amplitudes of some intermodulation products, for n=7 in (1) and an input signal given by (2).

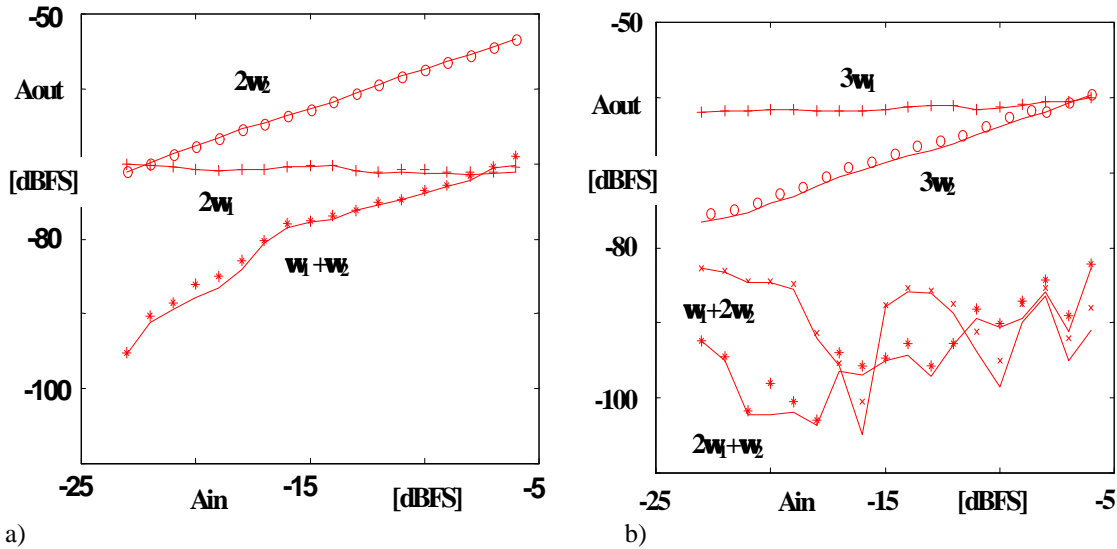


Fig.2. Measurement without dither. Amplitude of the second harmonic terms a) and third harmonic terms b) as a function of the amplitude of input signal 2. Lines are mean values, points are mean values plus standard deviation.

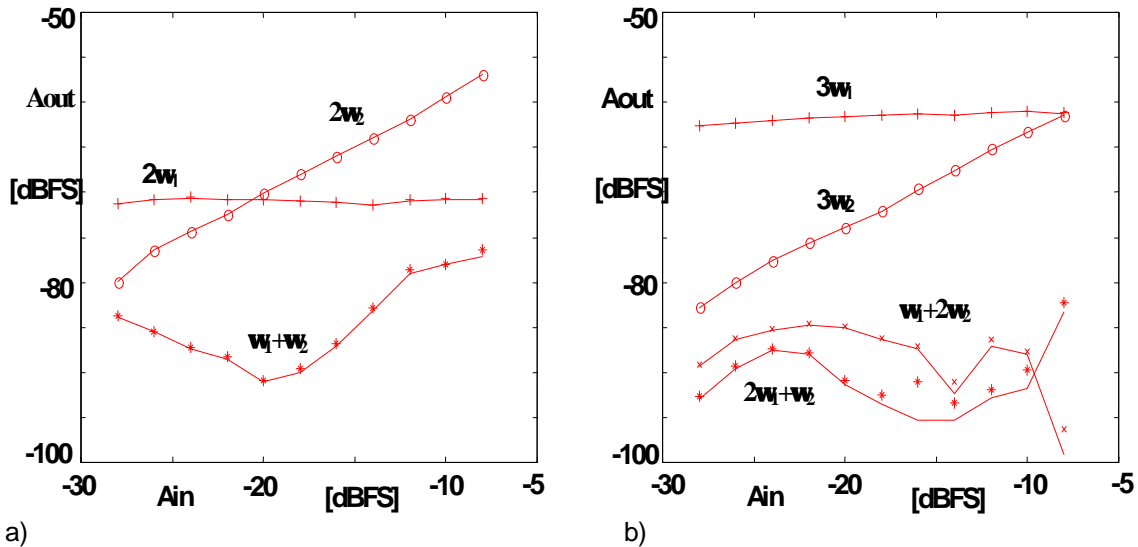


Fig.3. Measurement with dither. Amplitude of the second harmonic terms a) and third harmonic terms b) as a function of the amplitude of input signal 2. Lines are mean values, points are mean values plus standard deviation.

### 3 MEASUREMENT

We measured the amplitude of intermodulation products, to eliminate problems related to the purity of input signals. As the source of the input signal we use two PTS synthesizers. The used frequencies were 2000040 Hz and 2000030 Hz. One input signal, with frequency  $\omega_1$  has constant amplitude equal to ADC full scale minus 6dB (FS-6 dB), the other, with frequency  $\omega_2$ , had its amplitude changed by using an attenuator with 1 dB step. As the acquisition unit a home made digital receiver with HSP50016 was used [6]. The sampling frequency was 10 MHz, on-line filtering and sample decimation was 1000, data size 16 K samples. Every measurement was repeated 30 times to test the reproducibility. Fig. 2 shows experimental results for a 12 bits ADC (AD872). Mean value (lines) and the mean value plus standard deviation (points) are represented, to show the reproducibility. In Fig. 2a, which shows the second harmonic terms, the lines  $2\omega_1$  and  $2\omega_2$  are in reality the measurement of PTS harmonics. Since the amplitude of  $\omega_1$  is constant, also the amplitude of  $2\omega_1$  is constant. This PTS synthesizer has the second harmonics at -64 dBc. The amplitude of  $\omega_2$  linearly increases, so does the amplitude of  $2\omega_2$ . This PTS synthesizer has the second harmonics at -48 dBc. For an analysis of the ADC second harmonics term, the curve  $\omega_1 + \omega_2$  is important. It linearly increases in the interval [-16 dB, -6 dB]

according to the theoretical prediction. According to these measurement results, the level of the second harmonic terms produced by the ADC is -58 dB in the case of a full scale signal. The second order intercept point is +58 dBFS.

Fig. 2b shows the measurement results of the amplitudes of third harmonic terms. Curves  $3\omega_1$  and  $3\omega_2$  are the PTS harmonics, both synthesizers have the third harmonics about -54 dBc. Curves  $\omega_1+2\omega_2$  and  $2\omega_1+\omega_2$  would be important for the measurement of the third harmonic terms produced by the ADC. However, experimental results don't agree with the theoretical rules according to the intercept point. Stochastic changes appear as it would be the case of the presence of spurious signals and it is not possible to predict the changes in the amplitudes. We tried to repeat these measurements applying increased dither (fig.3) and the conclusion was: when the amplitude of the harmonics (with this ADC) is lower than -80 dBFS, the amplitude varies as if it was a spurious signal. The amplitude can not be predicted, and can even increase with decreasing input signals. The irregularity can be partly explained by the presence of high order polynomial terms (see Tab. 2) as it will be shown in next section (fig.4.d).

## 4 SIMULATION

Fig. 4 shows numerical simulation results for a 12 bits ADC affected by quantization and possessing a nonlinear transfer curve. The input signal is according to Eq. 3, amplitude A is -6 dBFS, amplitude B is plotted on the X-axis, it changes from -25 to -6 dBFS. Full scale saturation is not allowed. In the case of a), b), c) the non zero polynomial coefficients are:  $a_1=1$ ;  $a_2=a_3=0.001$ . Fig. 4a shows the result when no input noise or dither is used. Amplitude irregularities result from the existence of spurious components (quantization produces spurious when a sufficient level of input noise or dither is not used). In other cases, b), c), d) an input noise with rms of 1 LSB is used. Fig. 4b confirms the validity of the mth order law. The amplitudes of harmonics and intermodulation products increase according to the theoretical rules. The deviation from straight lines at low amplitudes is a result of the presence of noise, the noise level is in this case around -110 dBFS. Fig. 4c shows results obtained by imposing some harmonic distortion in the input signal. The levels of input signal harmonic distortion were:  $2\omega_1 = -64$  dBc,  $3\omega_1 = -54$  dBc,  $2\omega_2 = -54$  dBc and  $3\omega_2 = -54$  dBc. In this case the harmonic terms are dominated by the input signal content, but the intermodulation terms change according to the mth-order law. Fig. 4d shows the influence of higher order non zero coefficients, in this case we chose  $a_4=-0.0026$  and  $a_5=-0.0015$ . It can be seen that even for these small values of the high order coefficients, the mth-order law is no longer valid. The amplitude variation is dominated by the highest coefficients and some minima in the output amplitude can occur. This can explain the decrease in the output amplitude with increasing input amplitude, shown in experimental results of figs. 2 and 3.

## 5 DISCUSSION

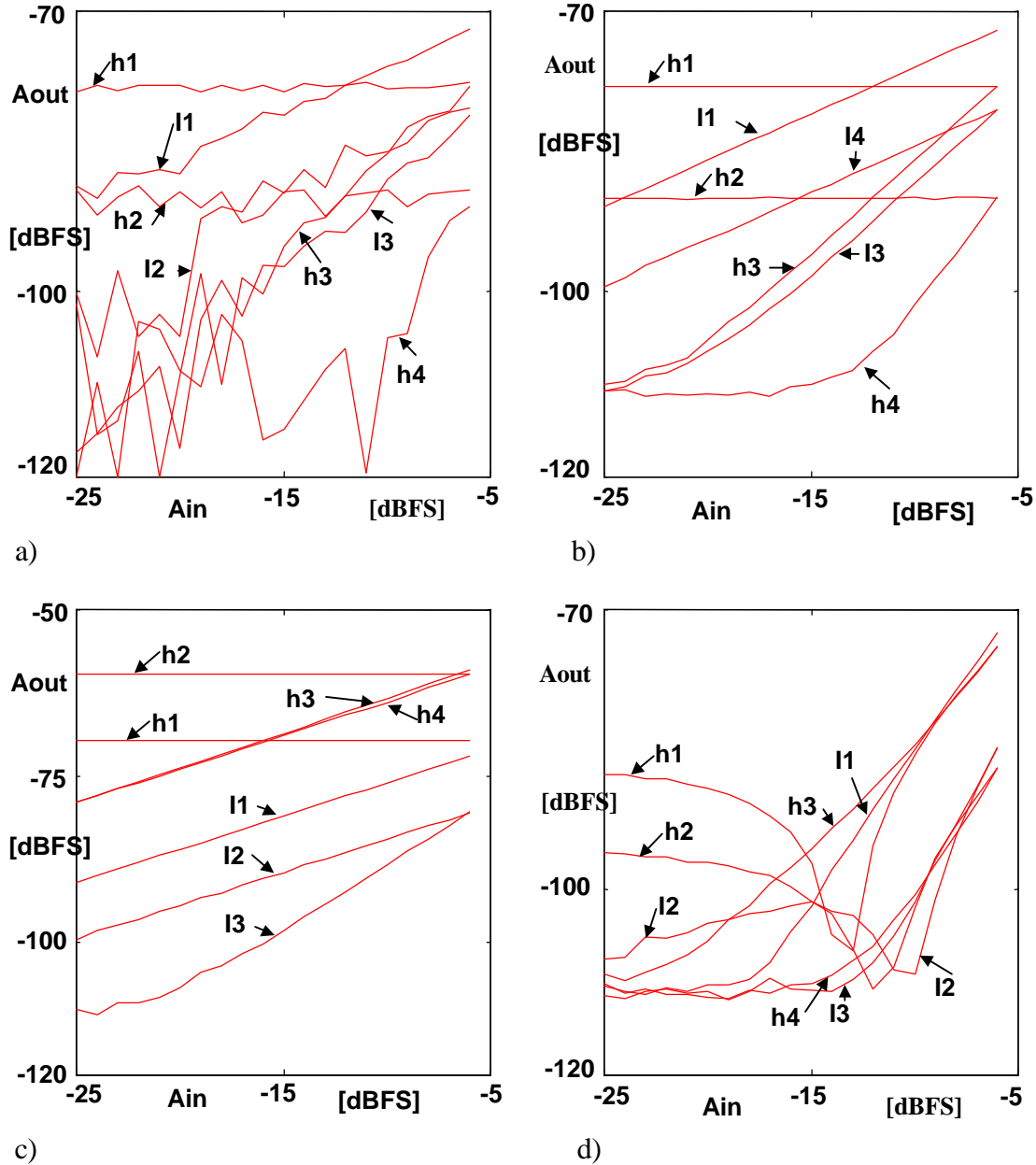
When analyzing ADCs behavior, one must consider many different types of nonlinearities with different influence. When measuring the harmonic and intermodulation distortion of ADCs, the influence of some nonlinearities must be minimized by using sufficient level of input noise or dither, see Fig. 2, 3, 4. The accuracy of measurements is limited not only by noise peak levels, but also by spurious, the influence of dither or input noise on the spurious free dynamic range, SFDR, is well known [7,8]. A more detailed analysis according to the best type of dither and different type of nonlinearities is missing. For example, in the case of undersampling applications the analysis of dynamic parameters of sample and hold circuit is missing.

The ADC's nonlinearities are mostly of higher order and we can not neglect the higher order coefficients of the transfer curve. These higher order coefficients can partly explain the irregularities of harmonics amplitudes (decrease in the amplitude with increasing amplitude of the input signal). The amplitude slope is dominated by the order of the last significant coefficient. So, the slope of the second harmonic term should be 2, 4, 6,... dB per dB of the input signal and the slope of the third harmonic term should be 3, 5, 7,... dB per dB of the input signal, see Tab.1. The mth-order law is valid only if the coefficients for  $n>3$  are so small that they can be neglected. The intercept points can only be used for those ADCs where the dominant nonlinearity is the transfer curve with order of 3 or less. Only in this case the amplitudes of harmonics can be predicted. The same is valid in the case of intermodulation distortions. Moreover, the amplitude of the intermodulation products and the sites where minimums occur depend on the amplitudes of both signals, see Tab.2. Small changes in the amplitude of one signal can substantially change the amplitudes of intermodulation products. This is the penalty for intermodulation distortion measurement. The advantage is that harmonic distortion of the input signal can be as high as -40 dBc.

Polynomial coefficients in (1) can not be estimated from the amplitude of a given harmonic. In fact, the contribution of the nth order coefficient to the amplitude of different harmonics diminishes with the order of the harmonics. For instance, the contribution of  $a_5$  for the amplitude of the 3<sup>rd</sup> harmonic is

$0.31a_5A^5$  (see table 1) and this value is 5 times bigger than its contribution for the amplitude of the 5<sup>th</sup> harmonic ( $0.062a_5A^5$ ). As a consequence, the INL computed from harmonic distortion [3] will be mostly inaccurate. This can explain also the problems with the scale mentioned in [3].

Other ADC nonlinearities such as the dynamic behavior of the sample and hold circuit produce also harmonic distortion. The influence of these nonlinearities must be analyzed independently of the influence of the transfer curve.



**Fig.4.** Simulation of the intermodulation distortions of a 12 bit ideal ADC with nonlinear transfer curve. Nonzero coefficients of the transfer curve: a) b), c)  $a_1=1$ ;  $a_2=a_3=0.001$ ; case d)  $a_1$  to  $a_3$  as in a) and  $a_4=-0.0026$ ,  $a_5=-0.0015$ . In cases b), c), d) the input noise with rms = 1 LSB is used. In the case c) the harmonic distortion of the input signal is supposed. Line markers:  $2\omega_1 = h1$ ;  $3\omega_1 = h2$ ;  $2\omega_2 = h3$ ;  $3\omega_2 = h4$ ;  $\omega_1 + \omega_2 = I1$ ;  $2\omega_1 + \omega_2 = I2$ ;  $\omega_1 + 2\omega_2 = I3$ .

The accuracy of our measurement results (Fig. 2,3) was not limited by noise peaks, they were about -105 dBFS, but the intermodulation distortion is dependent on more nonlinearities, not only by the transfer curve. In the case of this ADC, there is an influence of dynamic nonlinearities, the results for changed  $\omega_1$ ,  $\omega_2$  were rather different.

## 6 CONCLUSION

The amplitudes of the ADC's harmonic and intermodulation products can be predicted only in those cases where the low order nonlinearity of the transfer curve is dominant and coefficients of the order of 4 and higher can be neglected. The accuracy of measurement is limited not only by noise peaks but also by SFDR, and a sufficient level of dither or input noise must be used to decrease SFDR. High order nonlinearities of the ADC produce high order harmonics and intermodulation, but their amplitudes are mostly below SFDR and these products can be counted as spurious.

In our opinion, the following problems are rather neglected in the present ADC standards and the exchange of experience and the discussion should be useful:

- a) The influence of input noise or dither on SFDR results, and which type of dither should be used;
- b) The analysis of different type of ADC nonlinearities and their influence on SFDR and harmonic distortion;
- c) The definition of the difference between spurious and harmonics products, if some rules about harmonics amplitude or order are supposed.

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