Investigation of the Correction Factor for Ultrasonic Flow Meters

I. Gryshanova[1], A. Rak[2]

[1]National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", irgryshanova@gmail.com, Kyiv, Ukraine

[2]SE "Ukrmetrteststandard", Kyiv, Ukraine

E-mail (corresponding author): irgryshanova@gmail.com

Abstract

The main source of errors while applying modern ultrasonic flow measurement principle is the deviation of the actual velocity profile of the measured flow from the calculated one. If the velocity profile is known, the corresponding correction can be evaluated and considered during calibration. However, in practice, the distribution of velocities in the cross section of the pipeline differs from the theoretical one, which leads to errors of hydrodynamic origin.
To determine the flow rate of the measuring medium, it is necessary to transform the flow velocity averaged along the acoustic path to the velocity averaged for the cross section of the flow meter. To do this, use the hydrodynamic correction factor, which is a function of the Reynolds number. The inaccuracy of this factor is the largest component of the total error of ultrasonic flowmeters. This is due to the fact that velocity distribution (and hence the hydrodynamic factor) use dependences obtained on the assumption that measuring flow is axisymmetric and the trajectory of the ultrasonic beam lies in the plane passing through the pipeline axis. Nevertheless, most industrial flow media have a distorted profile due to installation effects, which are an integral part of any hydraulic system. As a result, the determined average flow velocity does not correspond to the real one. Therefore, the problem of studying the influence of flow non-symmetry on the value of the hydrodynamic correction factor is relevant.
The effect of distortion of the velocity profile on the measurement results of ultrasonic flowmeters was evaluated using theoretical dependences describing non-symmetric velocity profiles. For this purpose, functions based on the power law of velocity distribution in smooth pipes with the imposition of some influence function, which depends on the radial and angular distances from the observation point to the pipeline axis, were used. However, some dependencies can only be applied to approximate real flow profiles.
For velocity profiles that do not have axial symmetry, the only correct way to accurately estimate the flow rate is to reconstruct 2D velocity field using algebraic techniques. The implementation of one of these methods was performed based on the inverse Abel's transform.
For velocity profiles that do not have rotational symmetry around the axis of the pipeline, the value of the measured velocity will depend on the angle of orientation of the measuring plane relative to the diametrical plane of the flow meter. The calculation of the actual average flow velocity in the cross section of the meter was obtained from a specific mathematical dependencies describing velocity distribution by integration technique.
This research allows us to conclude that it is possible to calculate the performance of ultrasonic flowmeters under conditions of distorted non-symmetric flows at $Re>10^4$ with sufficient accuracy using computational hydrodynamics, integration based on Abel's transform, methods of theoretical research and mathematical processing.

1. Introduction

The main task of the analytical study for time-of-flight ultrasonic flowmeter is to create its mathematical model that describes the time of ultrasonic waves propagation through the measuring medium with its following transformation to fluid velocity averaged for meter's cross-section. The procedure is to reproduce the measuring process with real velocity profile. In solving this problem, it is necessary to determine those physical laws that have a direct or indirect impact on meter's performance. Based on research of these laws compensation ways and methods for errors arising in the course of measurement are defined. However, the variety of technical solutions used today by manufacturers of such devices and the lack of a single theoretical approach that would take into account the influence on the ultrasonic vibrations in real structures of industrial flows determined the practicability and relevance of this paper.

When determining the flow rate using an ultrasonic flow meter, it should be considered that the device does not directly measure the flow rate, but the average flow velocity of the controlled medium in the pipeline. Accordingly, the flow meter readings depend on the hydrodynamic characteristics of the flow in the measuring section. If the velocity profile is known, the corresponding correction can be calculated and taken into account when calibrating. In practice, the distribution of velocities in cross section differs from the calculated one, which leads to errors of hydrodynamic origin.

The known dependences [1-4] for mentioned corrections were obtained based on the assumption that the flow of the measuring medium is axisymmetric, and the trajectory of the ultrasonic beam lies in the plane passing through the axis of the pipeline. However, most industrial flow media have a distorted axial symmetry profile due to installation effects (such as spatial elbows) that are an integral part of any hydraulic system. In most cases, even the available straight pipe sections declared by the manufacturer before and after the meter are not enough to correct the axial asymmetry [3]. It should be noted that for accurate measurements mainly multipath ultrasonic meters are used. In such devices, the trajectories of the beams are located in planes disposed at some distance from the axis of the measuring section. Thus, the above facts require analysis and study of traditional formulas to determine the hydrodynamic factor in order to assess its reliability.

2. Problem description

The value of the hydrodynamic or correction factor *m* (Equation 1) is defined as the ratio of the velocity $\bar{u}$ averaged for pipe cross-section to the velocity $\bar{u}_l$ averaged for length of sound propagation path

$$m = \frac{\bar{u}}{\bar{u}_l}. \qquad (1)$$

The inaccuracy of the factor m is the largest component of the error for ultrasonic flowmeters, which can reach up to 5-10% and even more, especially in the lower part of the range [3]. This is because it's rather problematic to distinguish the ranges of applicability of one or another turbulence model for a correct representation of the velocity profile and other flow parameters for metrological purposes. Therefore, several semi-empirical formulas are used to describe the velocity distribution (and, consequently, the hydrodynamic factor), which only approximate the processes that take place. The asymmetry of the measuring flow also has a significant effect on the value of the hydrodynamic factor.

The effect of the axial velocity profile distortion on measurement results for applied ultrasonic meters can be estimated by theoretical dependences describing asymmetric velocity distribution. For the model representation of turbulent flow velocity profiles, the functions proposed by Salami [5] are used. They are based on the power law of velocity distribution in smooth pipes with superimposition of some influence function depending on radial *r* and angular θ distances from observation point to pipeline axis.

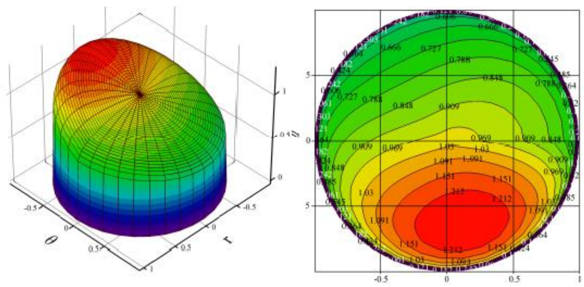
The advantages of using theoretical profiles are obvious, as they describe experimental flows. This allows us to estimate the effect of axial velocity profile distortion on the readings of ultrasonic flow meters without the application of field simulations. In addition, the velocity at any point can be calculated having no interpolation techniques.

Influence functions that create flow asymmetry and go to zero near the pipeline wall [5, 9] are described by trigonometric or radial (Equation 2) dependences:

$$u = (1-r)^{\frac{1}{n}} + m'r(1-r)^{\frac{1}{k}} \cdot f(\theta);$$
$$u = 1 + zr\sin(\theta)\ for\ r \le b, 0 \le b \le 1, \qquad (2)$$

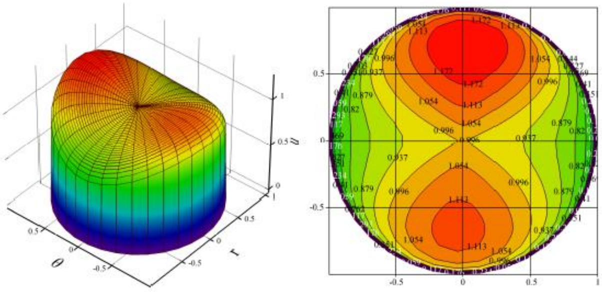
where $f(\theta)$ is certain defined function of θ; *b, n, k, m* are constants for specific velocity profile; *r* is a radial distance from the centre of the pipe to the observation point.
Constants and the form of influence functions for (Equation 2) obtained in [6].

The contours of the distorted velocity profiles can be divided into three groups. The first one includes profiles that have one peak (Figure 1). The second one - having two peaks (Figure 2). The third type includes more complex formations (Figure 3) that do not refer to the first or second groups.



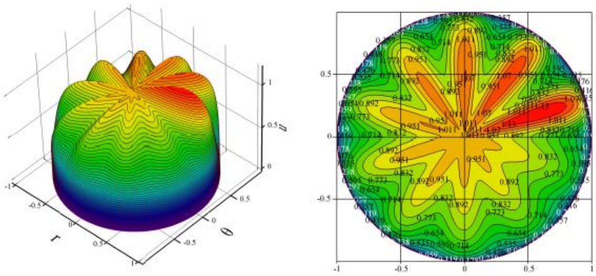
$$u(r,\theta) = (1-r)^{\frac{1}{9}} - \frac{0,5}{\pi} r(1-r)^{\frac{1}{4}} \times \theta(\sin(\theta))$$
$$r \le b,\ 0 \le b \le 1$$

Figure 1: Velocity profile with one peak.



$$u(r,\theta) = (1-r)^{\frac{1}{9}} + 0,6813 \cdot r(1-r)^{\frac{1}{9}} \times e^{-0,1\cdot\theta} \sin^2(\theta)$$
$$r \le b,\ 0 \le b \le 1$$

Figure 2: Velocity profile with two peaks.



$$u(r,\theta) = (1-r)^{\frac{1}{4}} + \frac{e^{0,15\pi}}{2} \cdot r(1-r)^{\frac{1}{9}} \times e^{-0,3\cdot\theta} \sin^2(5\theta)$$
$$r \le b,\ 0 \le b \le 1$$

Figure 3: Velocity profile of complex configuration.

Theoretical descriptions of velocity profiles represented by Equations 2 are used to estimate the distorted flows [7-9]. Therefore, it will be expedient to investigate these dependences for the reliability of their description of the actual measuring medium flows. These should be smooth functions for which the limit condition is zero flow rate on the walls of the measuring section. Differentiation of Equation 2 taking into account *f (θ)* by *θ* shows that among the fourteen profiles six do not have a continuous derivative at *θ*=0,2π. Similarly, by differentiating Equation 2 respectively to *r* there is also a discontinuity at *r* = 0. Thus, the theoretical velocity profiles proposed by Salami only resemble experimentally measured flow profiles [10]. This was also confirmed in [8]. Therefore, these dependencies can only be used to approximate real flow profiles.

For velocity profiles that do not have axial symmetry, the only correct way to accurately estimate the flow rate is to reconstruct 2*D* velocity field using algebraic techniques.

3. Theoretical estimation of hydrodynamic factor for axisymmetric flow

Let ξ_n be the distance from the n_{th} measuring plane to the origin; *D=2R* is the inner diameter of the measuring section; l_n is the distance between the emitter A_{n1} and the receiver B_{n1} for the n_{th} path (Figure 4). Consider vortex-free flow of the measuring medium in the channel in the positive direction of the z axis, i.e. $u = (0,0,u_z(x,y))$. We also assume that the transverse distribution of the velocity vector u_z has axial symmetry, therefore $u_z(x,y) = u_z(\sqrt{x^2+y^2}) = u_z(r)$, $r\epsilon[0,R]$, where $u_z(R) \equiv 0$ is boundary condition for walls of the measuring section.

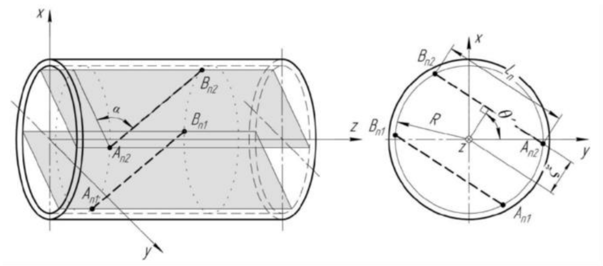


Figure 4: The scheme of multipath ultrasonic measuring section.

The difference in time for acoustic pulse propagation in n_{th} measuring plane $\Delta\tau(\xi_n) = \tau_{B_{n1}\to A_{n1}} - \tau_{A_{n1}\to B_{n1}}$ can be written as:

$$\Delta t(\xi_n) \cong \frac{2l_n \sin\alpha}{c_0^2} \bar{u}_z(\xi_n). \quad (3)$$

From the analysis (Equation 3) we can conclude that the average value of the flow velocity $\bar{u}_z(\xi_n)$ in the n_{th} measurement plane can be obtained on the basis of measurements $\Delta t(\xi_n)$, as:

$$\bar{u}_z(\xi_n) \cong \frac{c_0^2}{2l_n \sin\alpha} \Delta t(\xi_n). \quad (4)$$

In paper [11] it was shown that the average flow velocity $\bar{u}_z(\xi_n)$ in a plane at a distance ξ_n from the axis of the pipeline is associated with an unknown radial distribution of the real velocity vector $u_z(r)$ by integral Abel equation of the first kind for the function $u_z(r)$:

$$\bar{u}_z(\xi_n) = \frac{1}{\sqrt{R^2-\xi_n^2}} \int_{\xi_n}^{R} \frac{r}{\sqrt{r^2-\xi_n^2}} u_z(r)dr. \quad (5)$$

Expression (5) can be used for theoretical estimates of velocities in the corresponding measuring planes under the known radial distribution $u_z(r)$. The application of the transformation (5) is possible only if the function $u_z(r)$ is monotonic and non-increasing, and also limited on the interval [0, *R*]. Optimization of the estimation algorithm in the presence of measuring information only at two points 0 ⩽ *r*1 <*r*2 <*R* is proposed in the article [12].

The average flow rate of the measuring medium in the pipe cross section is expressed by the dependence:

$$\bar{u}_z(r) = \frac{2}{R^2}\int_0^R u_z(r)rdr. \quad (6)$$

Then, taking into account Equation 5 and Equation 6, the general expression for the hydrodynamic factor characterizing the relationship between the average flow velocity $\bar{u}_z(r)$ and the average beam velocity in the n_{th} plane, which is located at a distance ξ_n from the axis of the measuring section, $\bar{u}_z(\xi_n)$ will look like:

$$m = \frac{\bar{u}_z(\xi_n)}{\bar{u}_z(r)} = \frac{\frac{1}{\sqrt{R^2-\xi_n^2}}\int_{\xi_n}^R \frac{r}{\sqrt{r^2-\xi_n^2}} u_z(r)dr}{\frac{2}{R^2}\int_0^R u_z(r)rdr}. \quad (7)$$

The method of direct integrated estimates can be applied only if the two-dimensional velocity profile in the cross section has radial symmetry. But most fluid flows in round pipes are practically asymmetric. Distortion of the axial velocity profile is caused by any installation effect that is necessarily present in hydraulic systems. Non-axisymmetric flow profiles reduce the accuracy of ultrasonic flow measurements [13, 14].

4. Theoretical estimation of the hydrodynamic factor for a flow with distorted profile

To assess the effect of distortion of the axial velocity profile on the value of the hydrodynamic factor, we use a number of model profiles proposed by Salami [5].

In this situation, the only correct way to accurately estimate the flow rate value is a complete reconstruction of the 2*D* field of velocity distribution using algebraic reconstruction techniques or quasi-tomographic reconstruction, a method effective in case of incomplete data [11]. Implementation of one of such methods based on Abel's transform based on dependence inversion of Equation 5.

The inverse transform for Equation 5 can be rewritten as [11]:

$$\bar{u}_z(r) = -\frac{2}{\pi r}\frac{d}{dr}\int_r^R \xi\bar{u}_z(\xi)\sqrt{\frac{R^2-\xi^2}{r^2-\xi^2}}d\xi, \quad (8)$$

where the value of $\bar{u}_z(\xi_n)$ can be obtained by measuring the time difference $\Delta t(\xi_n)$ for the n_{th} measuring plane according to Equation 4.

Since the Salami model profiles [5] do not have rotational symmetry around the pipe axis, the value of the measured velocity $\bar{u}_z(\xi_n)$ will depend on the orientation angle of the measuring plane θ' relative to the diametrical plane of the measuring channel (Figure 4). To estimate the hydrodynamic factor under the condition of distorted axial profile, the Equation 7 will take the form:

$$m = \frac{\bar{u}_z(\xi_n,\theta\prime)}{\bar{u}_z(r,\theta)} = \frac{\frac{1}{\sqrt{R^2-\xi_n^2}}\int_{\xi_n}^R \frac{r}{\sqrt{r^2-\xi_n^2}} u_z(r,\theta\prime)dr}{\frac{2}{R^2}\int_0^R\int_0^{2\pi} u_z(r,\theta)d\theta rdr}, \quad (9)$$

where $\theta' = \mathrm{const}$ is the angle of measurement plane orientation; $\bar{u}_z(r,\theta) = \frac{2}{R^2}\int_0^R\int_0^{2\pi} u_z(r,\theta)d\theta rdr$ is flow velocity averaged in the cross section of the measurement channel.

6. Results and discussion

Results of theoretical investigation of hydrodynamic factor for different asymmetric flow profiles are represented on Figures 5-7.

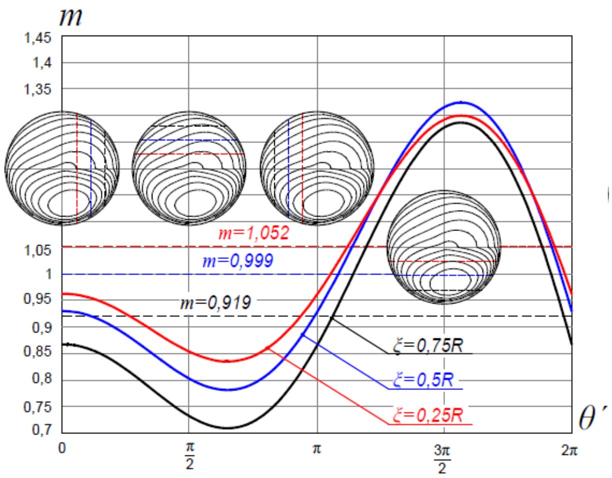


Figure 5: Estimation of hydrodynamic factor for profile with one peak.

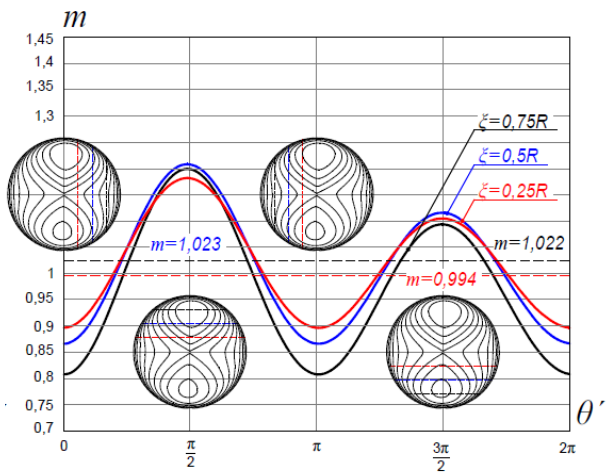


Figure 6: Estimation of hydrodynamic factor for profile with two peaks.

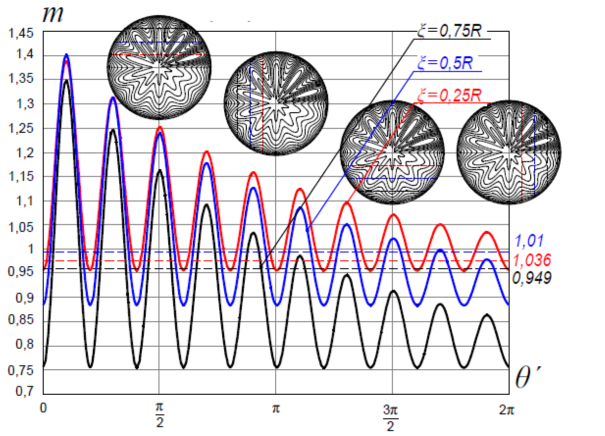


Figure 7: Estimation of hydrodynamic factor for profile of complex configuration.

It is possible to calculate the performance of ultrasonic flowmeters under conditions of distorted non-symmetric flows, where Reynolds numbers > 10^4 with sufficient accuracy using not only integration based on Abel's transform but also computational hydrodynamics. In particular, consider several examples of assessing the impact of asymmetry caused by the most well-known installation effects.

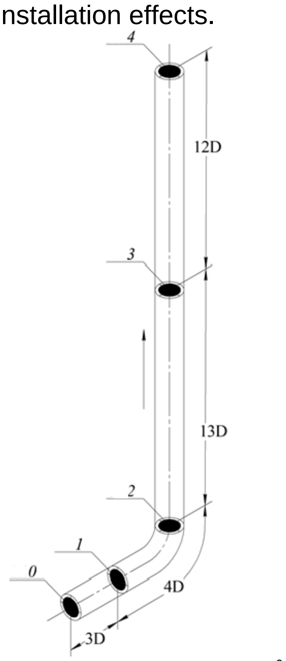


Figure 8: The scheme of 90^0 bend.

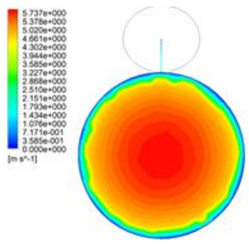


Figure 9: Velocity distribution for section 1

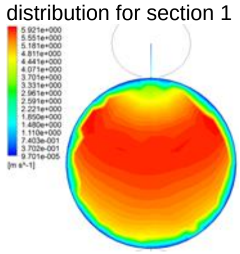


Figure 10: Velocity distribution for section 2.

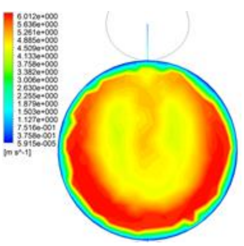


Figure 11: Velocity distribution for section 3.

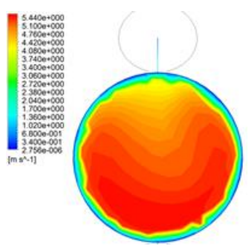


Figure 12: Velocity distribution for section 4.

Profiles after 90^0 bend (Figure 8) on Figures 10 - 12 have an asymmetric distribution, and therefore the measurement results will depend on the angle of orientation of the measuring chord relative to the axis of the pipeline. The angle of rotation is counted from the vertical axis clockwise with a discrete step of 45 °.

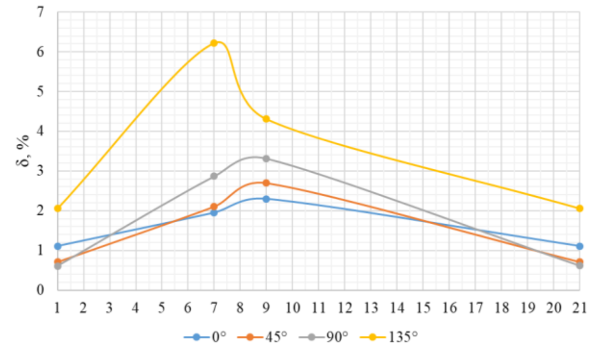


Figure 13: Relative measurement error depending on the orientation angle after 90^0 bend.

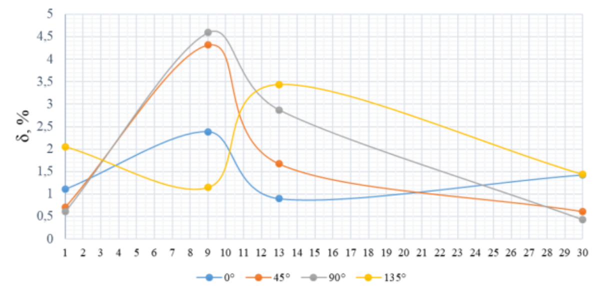


Figure 14: Relative measurement error depending on the orientation angle after two 90^0 bends located in one plane.

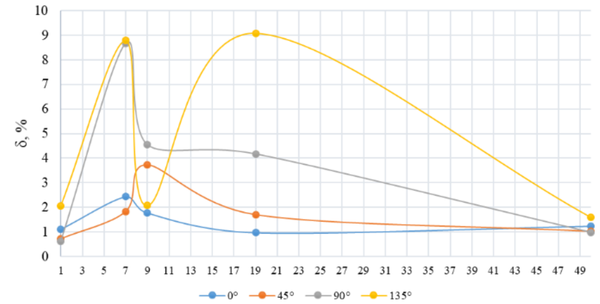


Figure 15: Relative measurement error depending on the orientation angle after two spatial 90^0 bends.

From the analysis of the received graphic dependences in Figures 13-15, it can be concluded that for considered profiles of asymmetric flows the error of determination of hydrodynamic factor in some cases can reach 10% and decreases with distance of control section from installation effect relative to the axis and diametrical plane of the channel.

7. Conclusion

For single-path designs of ultrasonic meters, the minimum error in determining the hydrodynamic factor can be achieved by placing the measuring chord in the diametrical plane. Given the fact that during operation, the direction of orientation of the asymmetry in the flow is difficult to predict and it may change during operation, a clear recommendation on the angle of the measuring planes relative to the diametrical plane is impossible. Therefore, for single-path ultrasonic meters, it is necessary to specify the value of the hydrodynamic correction factor experimentally.

A significant increase in the accuracy of flow measurement with distorted flow profiles can be achieved by using multipath designs of meters, several measuring planes of which are located at certain distances from the axis of the measuring section.

Thus, there are several ways to increase the accuracy of ultrasonic flow measurement:
- increase in the number of measuring chords, and as a consequence - the amount of measuring information, as well as to some extent compensate for the effect of asymmetry in their symmetrical placement relative to the axis of the measuring section. The advantage of this method is increasing the accuracy of measurement, the disadvantage is increasing the complexity of the design and cost of the device;
- creation of appropriate parameters of the measuring section, which in turn will create clearly defined parameters of the flow velocity distribution. The advantage of this method is not only an increase in accuracy, but also the preservation of performance characteristics (cost and energy consumption), the disadvantages are a significant increase in the value of hydraulic resistance.

References

[1] Gryshanova I, “The improved ultrasonic flow measuring method”, in *Proc. of the ASME Fluids Engineering Division Summer Conference* (FEDSM 2008), pp 337–340, 2009.

[2] Gryshanova I, “A computational investigation of flow meters”, in *Proc. of 15th International Flow Measurement Conference* (FLOMEKO 2010), pp 888–895, 2010.

[3] Gryshanova I, Korobko I, “How to improve accuracy of existing ultrasonic water meters”, in *Proc. of Fluids Engineering Division FEDSM* (American Society of Mechanical Engineers), 2, 2021

[4] Kremlevsky P P, *Flow meters and quantity meters* (Leningrad, Russia, Energy Publ. Co.), 1989.

[5] Salami L A, “Application of a computer to asymmetric flow measurement in circular pipes”, *Trans. Inst. Meas. Control,* **6**, pp197–206, 1984.

[6] Salami L A, “Velocity Area Methods for asymmetric profiles” *University of Southhampton Interim Report V,* 1972.

[7] Walus S, “The use of the ultrasonic flowmeter in the conditions other than normal”, in *Proc. of FLOMEKO ed H S Stephens et al* (University of Melbourne), pp 171–6, 1985.

[8] Frank S, Heilmann C, and Siekmann H E, “Point-velocity methods for flow-rate measurements in asymmetric pipe flow”, *Flow Meas. Instrum*., **7**, 201–9, 1996

[9] Moore P I, Brown G J and Stimpson B P, “Modelling of transit time ultrasonic flowmeters in theoretical asymmetric flow”, in *Proc. of FLOMEKO* (Salvador, 2000), 2000.

[10] Salami L A, “Errors in the velocity-area method of measuring asymmetric flows in circular pipes”, *University of Southampton Interim Report I*, 1971.

[11] Rychagov M N, Tereshchenko S A, Dean B, Lynnworth L, “Multipath Flowrate Measurements of Symmetric and Asymmetric Flows”, in *Proc. of the 1st World Congress on Industrial Process Tomography* (UK, Buxton), pp 438-443, 1999.

[12] Nikolayeva N N, Ruchkin S V, Rychagov M N, Yagola A G, “Chislennoye modelirovaniye zadachi dvumernoy rekonstruktsii aksial'nykh osesimmetrichnykh profiley skorosti”, *Vychislitel'nyye metody i programmirovaniye*, **Vol 6**, pp 9–16, 2005.

[13] Hilgenstock A, Heinz M and Nath B, «Numerical flow simulation as a tool for developing and calibrating ultrasonic flow meters», in *Proc. of the 8th Int. Conf. of flow measurement, FLOMEKO'96* (Beijing, China), pp 1 – 6, 1996.

[14] Fleury G, «Reconstruction de profils de vitesse a partir de donnees lacunaires et optimisation d'instrument — Application a la débimetrie ultrasonore», *Dissertation* (Orsay, France: Université de Paris-Sud, École Supérieure d'Électricité), 1994.