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Evaluation of Uncertainty in Measurement of the Super-heated Steam Density based on Monte Carlo Method

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Abstract

In this paper, Monte Carlo method (hereinafter referred to as MCM) is used to evaluate the uncertainty of super-heated steam density. Through more than 800k simulation tests using the MCM ALCHIMIA software, the probability density function, coverage probability and coverage interval of the super-heated steam density are obtained, and the graphic shape of probability density function is analysed. In the process of evaluating the measurement uncertainty of superheated steam density, it is not rigorous to blindly define its probability distribution as uniform distribution. At the same time, when evaluating the uncertainty of superheated steam density, the use of gum has inherent shortcomings, and the use of MCM can get the real situation more accurately.

1. Introduction

Super-heated steam is an important energy carrying medium, which is widely used in power, medicine, food processing and other fields. Therefore, the metrology of super-heated steam is of particular importance. At present, the domestic flow sensors mainly used for super-heated steam metrology are vortex flowmeter and differential pressure flowmeter. At the same time, the metrology system is composed of temperature sensor, pressure sensor and flow totalizer to measure the mass or energy of super-heated steam.

Since the flow sensor can only provide volume signal, it is also necessary to measure the working temperature and pressure of super-heated steam and calculate the density of super-heated steam, so as to obtain the mass of super-heated steam. Therefore, in the steam measurement system using flow totalizer, the uncertainty introduced by super-heated steam density is an important part of the whole measurement model.

2. Comparison between GUM and MCM methods

2.1 A brief introduction to the calculation formula of super-heated steam

The calculation formula of super-heated steam density is determined according to appendix A 2.3 of JJG1003-2016 flow totalizer and IAPWS-IF97 (hereinafter referred to as IFC formula), and the content of the formula is as follows (hereinafter referred to as Formula 1)

$$
\begin{cases}
\rho = \dfrac{1}{0.00317 v_r} \\
v_r = A - B - C - D - E - F - G - H + I - J - K \\
A = \dfrac{4.260321148 T_r}{P_r} \\
B = 0.066703759 x^{13} \\
C = 1.388983801 x^{13} \\
D = 2P_r \left(0.08390104328 x^{18} + 0.02614670893 x^{2} - 0.03373439453 x\right) \\
E = 3P_r^{2} \left(0.4520918904 x^{18} + 0.1069036614 x^{10}\right) \\
F = 4P_r^{3} \left(-0.5975336707 x^{25} - 0.08847535804 x^{14}\right) \\
G = 5P_r^{4} \left(0.5958051609 x^{32} - 0.5159303373 x^{28} + 0.2075021122 x^{24}\right) \\
H = 6P_r^{-7} \left(0.006552390126 x^{24} + 0.0005770218649 x^{14}\right) \begin{pmatrix} P_r^{-6} - 0.8532322921 x^{54} \\ +0.3460208861 x^{27} \end{pmatrix}^{-2} \\
I = 11 \left(\dfrac{P_r}{P_l}\right)^{10} \begin{pmatrix} 193.6587558 - 1388.522425 x + 4126.607219 x^{2} - 6508.211677 x^{3} \\ +5745.984054 x^{4} - 2693.088365 x^{5} + 523.5718623 x^{6} \end{pmatrix} \\
J = 4P_r^{-5} \left(0.1190610271 x^{12} - 0.09867174132 x^{11}\right)\left(P_r^{-4} - 0.4006073948 x^{14}\right)^{-2} \\
K = 5P_r^{-5} \left(0.1683998803 x^{23} - 0.05809438001 x^{18}\right)\left(P_r^{-5} - 0.08636081627 x^{19}\right)^{-2} \\
T_r = \dfrac{T}{647.3} \\
P_r = \dfrac{P}{22.12} \\
P_l = 15.74373327 - 34.17061978 P_r + 19.31380707 P_r^{2} \\
x = e^{0.7633333333(1 - T_r)}
\end{cases}
$$

2.2 Issues when using GUM to evaluate the uncertainty of super-heated steam

When using GUM to evaluate the uncertainty of super-heated steam density (here, the uncertainty evaluation is carried out by means of Class B method), it is necessary to know the probability distribution type of super-heated steam density. However, through the analysis of the measurement model in Formula 1, it can be found that the density of super-heated steam is obtained by a series of complicated calculations from the two inputs of temperature and pressure. When GUM is used to evaluate the uncertainty, it is difficult to determine the corresponding sensitivity coefficient by manual calculation. At present, the general evaluation method is to assume the probability distribution of super-heated steam as uniform distribution. However, this method is only a new hypothesis, and the reliability of its calculation results needs to be verified.

2.3 Advantage of using MCM to evaluate the uncertainty of super-heated steam

MCM is a method to obtain the probability density function of the objective function according to the known measurement model by using a large number of random numbers. Therefore, the probability density function can be accurately obtained only by correctly putting the measurement model formula of super-heated steam density. Therefore, MCM is more suitable for evaluating the uncertainty of super-heated steam density.

3. Evaluation of uncertainty of super-heated steam density by GUM (Class B)

3.1 Summary

3.1.1 Measurement environment

Assuming that the medium in the pipeline to be measured is super-heated steam, the measurement range of the pressure transmitter is (0~2.5) MPa, the accuracy level is 0.2, the type of the temperature sensor is Pt100, and the accuracy level is B.

3.1.2 Measurement results

Through pressure transmitter and platinum thermal resistance, and measured by flow totalizer, the absolute pressure in the pipeline is 2.000 MPa, and the half-width is 0.005 MPa, assuming it obeys uniform distribution. The temperature is 230 ℃ (503.15K), assuming that it obeys uniform distribution. According to the requirements of JJG 229-2010, when measuring at 230℃, its maximum permissible error is ±(0.30+0.005 t) = ±1.45℃, and its half width is 1.45 K.

3.1.3 Other instructions

For the convenience of calculation, the pressure in 3.1.2 is expressed by absolute pressure, and the influence of uncertainty introduced by flow totalizer is ignored.

3.2 Determine the half width of super-heated steam density

Because the density of superheated steam is direct proportional to the pressure change and inverse proportional to the temperature change, within the range of (2±0.005) MPa and (503.15±1.45) K, the density of super-heated steam varies from (9.427230 ~ 9.563228) kg/m^3, and its half width is 0.067999 kg/m^3. If it obeys uniform distribution, the standard uncertainty of super-heated steam density at 230℃ and 2MPa, u_{GUM} = 0.067999/1.732 = 0.0393 kg/m^3.

4. Evaluation of density uncertainty of super-heated steam by MCM

4.1 Summary

4.1.1 Measurement environment

Assuming that the medium in the pipeline to be measured is super-heated steam, the measurement range of the pressure transmitter is (0~2.5) MPa, the accuracy level is 0.2, the type of the temperature sensor is Pt100, and the accuracy level is B.

4.1.2 Measurement results

Through pressure transmitter and platinum thermal resistance, and measured by flow totalizer, the absolute pressure in the pipeline is 2.000 MPa, and the temperature is 230℃(503.15K).

4.1.3 Other instructions

For the convenience of calculation, the pressure in 4.1.2 is expressed by absolute pressure, and the influence of uncertainty introduced by flow totalizer is ignored.

4.2 Measurement model

Refer formula 1.

4.3 Set the probability density function of each input quantity (PDF)

4.3.1 PDF of pressure P

Because the accuracy level of the pressure transmitter is 0.2, the measuring range is (0~2.5) MPa, and its probability distribution obeys uniform distribution, the half-width is 0.005MPa.

4.3.2 PDF of temperature T

As the accuracy of industrial platinum thermal resistance thermometer is grade B, its probability distribution obeys uniform distribution. According to the requirements of its verification regulations, when measuring at 230℃, its maximum permissible error is ±(0.30+0.005 t) = ±1.45℃, and its half width is 1.45 K.

4.4 Select Monte Carlo test number

According to the requirements of JJF1059.2, the number of tests M should be greater than 104 times of 1/(1-p), and if p=95.45%, then M should be greater than 2.1978×105. As this evaluation is calculated by professional software, 800,000 trials are selected.

4.5 Sampling of probability distribution of input quantity and calculation of model value

The professional software used in this evaluation is MCM Alchimia (hereinafter referred to as software). After running the software, enter the main interface of the software, as shown in Figure 1.

Figure 1: MCM Alchimia software main interface.

According to the six steps on the left side of the software interface, you can get the evaluation results after setting them step by step, shown in the following Figures.

Figure 2: PROJECT INFO setting.

Enter OVERHEAT STEAM here in Title, and other contents are omitted.

Figure 3: MODEL DEFINITION setting.

After the equations of formula 1 are in sequence brought in.Enter the contents obtained into Equations group, and click Update parameters in the upper right corner, then all variables in the formula will appear. Set the units and descriptions for the three variables respectively. Since the unit of pressure in formula 1 is MPa, but the unit of pressure in software is Pa, bar is used here instead of MPa, and description is given, as shown in Figure 3.

Figure 4: Setting of SIMULATION DATA and setting of input quantity T.

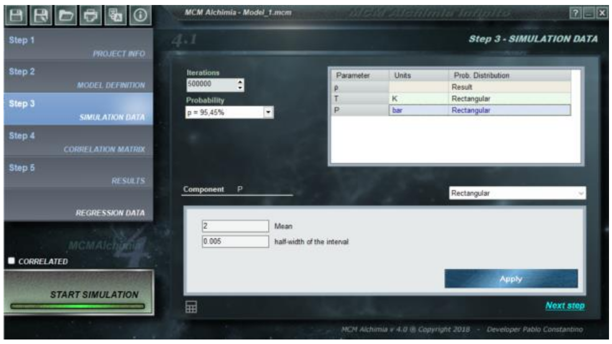

Figure 5: Setting of SIMULATION DATA and setting of input quantity T.

Set the number of iterations to 500,000, the inclusion probability to 95.45%, and the probability distribution of the input quantity T to be Rectangular (rectangular distribution) with a value of 503.15 and a half-width of 1.45, then set the probability distribution of the input quantity P to be Rectangular, with a value of 2 and a half-width of 0.005, as shown in Figure 4 and Figure 5.

Figure 6: CORRELATION MATRIX setting.

In step 4, CORRELATION MATRIX setting, there is no need to set it here, but CORRELATED cannot be checked, because the input quantities P and T are not related to each other. After setting, click the START SIMULATION button at the bottom left and the calculation results are obtained, as shown in Figure 7.

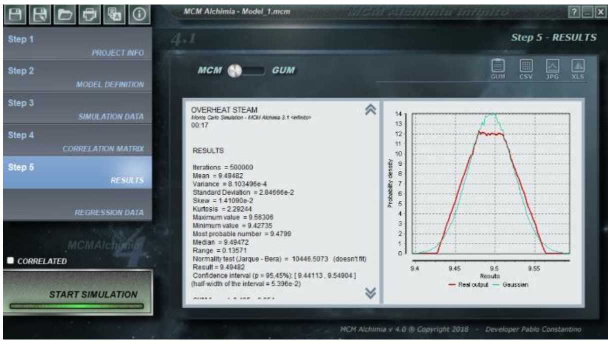

Figure 7: Calculation results.

4.6 Estimate value of output ρ and its standard uncertainty and inclusion interval

According to figure 7, the estimate value of output ρ is 9.49482 kg/m^3, the standard uncertainty is 0.0540 kg/m^3, and the included interval is (p = 95.45%): [9.44113 kg/m^3, 9.54904 kg/m^3]

4.7 Report results

MCM is used to calculate the density of super-heated steam with the temperature of 503.15K and absolute pressure of 2 MPa for 500,000 times, and the evaluation results of uncertainty as follows: ρ = 9.49482kg/m^3, u (ρ) = 0.0540kg/m^3, the shortest 95.45% inclusive interval = [9.44113 kg/m^3, 9.54904 kg/m³]。

5. Verification and analysis of the results

5.1 Verify the validity of the software calculation formula

Due to the complexity of the calculation formula of super-heated steam, inevitably there will be various errors when the formula 1 is converted into the software calculation formula. The super-heated steam density table is used to verify the validity of the software calculation results, the super-heated steam density is 9.4948 kg/m^3 at the absolute pressure of 2 MPa and the temperature of 230℃ (see figure 8 below).

附录三　过热蒸汽密度表 (单位：ρ=Kg/m3)

P MPa	t（℃）						
	150	170	190	210	230	250	270
0.10	0.5165	0.4924	0.4705	0.4506	0.4324	0.4156	0.4001
0.15	0.7782	0.7413	0.7077	0.6775	0.6498	0.6246	0.6010
0.20	1.0422	0.9918	0.9465	0.9056	0.8683	0.8341	0.8026
0.25	1.3087	1.2443	1.1867	1.1348	1.0877	1.0445	1.0048
0.30	1.5779	1.4988	1.4284	1.3652	1.3079	1.2556	1.2076
0.40	2.1247	2.0138	1.9164	1.8295	1.7513	1.6801	1.6150
0.50	2.6643	2.5376	2.4108	2.2989	2.1983	2.1079	2.0251
0.80		4.1357	3.9370	3.7383	3.5651	3.4106	3.2712
1.10		5.8404	5.5368	5.2331	4.9728	4.7451	4.5424
1.40			7.1526	6.7895	6.4263	6.1145	5.8411
1.70			8.9040	8.4177	7.9315	7.5224	7.1692
2.00				10.017	9.4948	8.9727	8.5293
2.50				13.005	12.2507	11.4962	10.8747
3.00					15.0180	14.1742	13.3307
3.50					18.1610	17.0378	15.9144
4.00						19.8010	18.6463
4.50						23.0200	21.5564
5.00						26.5150	24.6731

Figure 8: Super-heated steam density table.

The result calculated by MCM Alchimia software is 9.49482kg/m^3, and the relative error between them is 2.1ppm. Therefore, it can be considered that the density result calculated by the software is correct.

5.2 Result analysis

By observing the calculation results shown in Figure 7, we can see that under the current working condition, the PDF of ρ is an approximate asymmetric trapezoid, rather than the uniform distribution mentioned in 2.2 of this paper.

5.3 Comparison with GUM evaluation results

To sum up, it can be seen that u_{GUM} = 0.0393 kg/m^3 and u_{MCM} = 0.0540 kg/m^3, and the relative error between them is 27.2%. It can be seen that the final results of the two methods are not consistent, so it is not recommended to use GUM to evaluate the uncertainty of super-heated steam density.

6. Conclusion

By using MCM to evaluate and analyze the density of super-heated steam, it can be found that the probability density function of super-heated steam does not obey the uniform distribution, nor does it obey the trapezoidal distribution, which is an approximate asymmetric trapezoid under the working conditions of 230℃ and 2MPa. Therefore, in the process of evaluating the measurement uncertainty of super-heated steam density, it is not rigorous enough to blindly define its probability distribution as uniform distribution. At the same time, when evaluating the uncertainty of super-heated steam density, GUM has inherent shortcomings, while MCM can get the real situation more accurately.

References

1. IAPWS-IF97: *The Industrial Formulation for the thermodynamic properties of water and steam.*
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3. JJG 1003-2016: *Verification regulation of Flow Integration Meters*.
4. JJF 1059.2-2012: *Monte Carlo Method for Evaluation of Measurement Uncertainty*.