Effect of Different Conditions on the Boundary Layer Transition of Sonic Nozzle

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Abstract

The boundary layer transition of sonic nozzle is affected by many factors, including Reynolds number, macro structure (throat diameter), meso-micro surface structure (wall roughness, waviness), turbulence intensity, wall heat transfer and so on. Therefore, in order to analyse the influences of macrostructure, meso-micro surface structure and turbulence intensity on boundary layer transition, the C_d and geometric dimension (throat diameter, *d*, average roughness, *Ra*, maximum roughness, *Rz*, and waviness, *wa*) of sonic nozzles with throat diameter of 1.919 mm, 3.808mm, and 7.453 mm were experimental investigated and measured. Furthermore, a series of CFD simulations through the transition SST model for axisymmetric nozzles were established to research the variation law of boundary layer transition and C_d of the sonic nozzle when Reynolds numbers ranges from 4.6×10^4 to 4.7×10^7 and different turbulence intensity (*Tu*) and meso-micro surface structure (*Ra*, *Rz* and *wa*). The validity of the simulation model was confirmed by the experimental data of National Institute of Metrology of China (NIM). Finally, the relationships between the boundary layer transition and *Tu*, *Ra*, *Rz* and *wa* were obtained.

1. Introduction

The critical flow venturi nozzles (known as sonic nozzles) are frequently used as master meter to calibrate other kinds of gas meters and has been applied to high pressure natural gas measurement, due to their high accuracy, excellent repeatability, and absence of moving parts. The flow field in sonic nozzle can be divided into two areas near-wall region and the multi-dimensional core region. And, there is a laminar, transition or turbulence boundary layer near the wall. The discharge coefficient C_d is key parameter of the flow characteristic of sonic nozzles, which is defined as the ratio is a dimensionless ratio of the actual flow-rate to the ideal flow-rate. With the increase of inlet pressure or Reynolds number, the boundary layer near the wall begins to transition from laminar flow b to turbulence flow, and the C_d has a reduced "jump" in this transition process. The decrease of C_d caused by boundary layer transition which might affect the measurement accuracy. The boundary layer transition of sonic nozzle is affected by many factors, including Reynolds number (Re), macro structure (throat diameter), meso-micro surface structure (surface roughness, waviness), turbulence intensity, wall heat transfer and so on.

In 1964, Stratford[1] found that the boundary layer transition from laminar to turbulent taking place at the Reynolds number of between 1×10^6 and 2×10^6 through experimental study. In 1992, Demetriades[2] measured the Reynolds number (Re_{tr}) of the boundary layer transition under different roughness by using the hot film anemometer, and found that the Re_{tr} decreased with the increase of surface roughness. In 2005, the newest version of ISO 9300[3] made preliminary requirements for the geometry structure of sonic nozzles based on the existing theoretical formulas and experimental research results, and divided them into two types: ‘normally machined’ and ‘accurately machined’ according to different surface roughness levels. In 2009, Mickan[4] obtained that the measurement deviation of the C_d reached 0.6% due to the existence of surface roughness. In 2015, Ishibashi[5] also realized the influence of surface roughness on the development characteristics of boundary layer. In order to avoid the transition of boundary layer, ultra-precision sonic nozzle was selected to ensure that the fluid flow is always in the region of laminar boundary layer. In the same year, Lavante[6] used the intermittent factor γ for the first time to predict the influence of free turbulence intensity *Tu* on the boundary layer transition, but he did not consider the additional effect for wall roughness. In 2015 and 2016, Wang and Ding [错误!未找到引用源。-错误!未找到引用源。] found that the influence of surface roughness in the turbulent boundary layer is larger than in the laminar boundary layer and proposed the dimensionless relative roughness should be considered rather than an absolute roughness value. In 2018, Li[9] studied the influence mechanism of throat diameter on the boundary layer flow characteristics of sonic nozzle, and pointed out the C_d and the Re_{tr} are both influenced by the throat diameter.

In addition, the use of micro structure parameters varies from country to country. At present, China and North America often adopt the average surface roughness *Ra*, while European countries generally adopt the maximum surface roughness *Rz*. At the same time, as a very important parameter in fluid mechanics, equivalent roughness K_s. It need to be studied and discussed what is the relationship between K_s, *Ra* and *Rz*, and which parameter can be used to more reasonably and intuitively evaluate the influence of micro surface structure on the development of boundary layer.

This study focused on the effect of different conditions on the boundary layer transition of the sonic nozzle. The C_d of three sonic nozzle with throat diameter of 1.919mm, 3.808 mm, and 7.453 mm were investigated at the pVTt facility in National Institute of Metrology of China (NIM). In view of the complexity of the boundary layer transition, a good transition model for investigating the boundary layer transition is necessary. The SST (Shear-Stress Transport) k-ω model of the transition has been proposed as early as 1994 by Menter. This paper choose the transition SST model to analyse the influence of the meso-micro surface structure and turbulence intensity on the discharge coefficient and boundary layer transition. Finally, the *relationship between Ra, Rz, wa and* K_s, and the transition characteristics of the boundary layers of sonic nozzles were obtained by the experimental data and simulation results.

2. Measurement results of surface structure and C_d of sonic nozzle

2.1 Results of the meso-micro surface structure

Three sonic nozzle with a design throat diameter of 1.919 mm, 3.808 mm and 7.453 mm were selected and their meso-micro surface structures were measured in detail. Since the entrance to the throat of the sonic nozzle is an arc segment, if the structure is not damaged, the location for measuring the meso-micro surface structures will be concentrated in the upstream and near the throat of the sonic nozzle. Similar to the method of macro structure measurement[10], the distance 1*d* upstream of the throat was measured, and the sampling length was 2.5 mm. 8 points were measured on each section at an interval of 45º. The average value was taken as the surface waviness and roughness of the sonic nozzle.

Various surface roughness and waviness parameters were measured with the Taylor Hobson high-precision surface roughness instrument of NIM. The device is shown in Figure 1.

Figure 1: High-precision surface roughness instrument.

The surface waviness and roughness of each sonic nozzle shall be measured at least 10 times, and the average value shall be taken as the result. The surface structure of one measurement was shown in Figure 2.

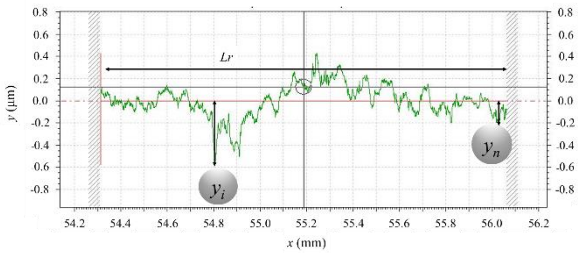

Figure 2: The oscillogram of meso-micro surface structure.

The measurement results of the meso-micro surface structure for three sonic nozzle were shown in table 1. The surface waviness of the 3.808 mm sonic nozzle was not actually measured

Table 1: The measurement results of the meso-micro surface structure for 7.453 mm sonic nozzle.

Throat diameter *d* (mm)	Surface waviness *wa* (μm)	Average surface roughness *Ra* (μm)	Maximum surface roughness *Rz* (μm)
1.919	47.00	0.17	1.10
3.808	\	0.06	0.60
7.453	5.20	0.05	0.44

2.2 Results of the C_d

The experimental C_d of this sonic nozzle at different Reynolds number were conducted by two pVTt gas flow standard facilities constructed in 1986 and 2014 respectively at NIM, China[9]. One apparatus with the expanded uncertainty of

0.10%~0.20% (k=2) is used to calibrate the C_d with the flow rates range from 1 m³/h to 1138 m³/h at the stagnation pressure of 0.1 MPa. Another apparatus utilizes a dry compressed air to calibrate the C_d covering flow range extending from 0.019 kg/h to 1367 kg/h, and the stagnation pressure range from 0.1 MPa to 2.5 MPa. The expanded uncertainty of apparatus could be 0.06% (k=2), while the expanded uncertainty of C_d was 0.08% (k=2). The results were shown in Figure 3, in which a total of 79 measuring points of the C_d were plotted. The 'normally machined' and 'accurately machined' equations of ISO 9300 respectively expressed as "ISO 9300-n" and "ISO 9300-a".

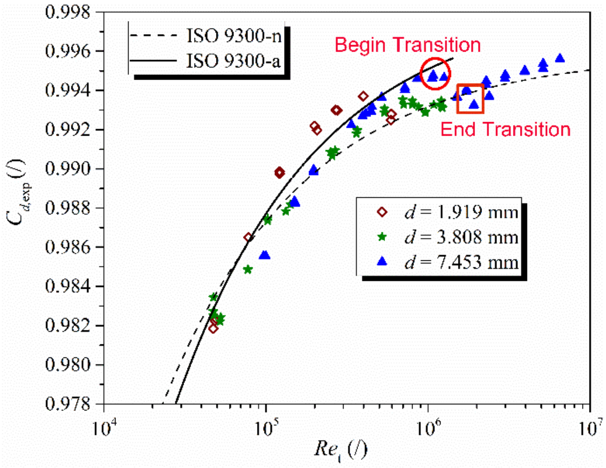

Figure 3: The values of the C_d for three sonic nozzle.

In Figure 3, it implied that the relationship between C_d and the throat Reynolds numbers Re_t was dependent on different boundary layer conditions. For the laminar boundary layer, the C_d significantly increases with the increasing of Re_t. For the transition boundary layer, the C_d had a reduced 'jump down', while in the turbulent boundary layer, the C_d also showed a tiny increasing tendency. Besides, it also obtained conclusion that the transition position of 1.919 mm sonic nozzle is at the Re_{tr} of 3.9×10⁵ rather than 1×10⁶ ~ 2×10⁶.

By fitting the experimental data of each nozzle with the reference of 'S-CURVE' by Ishibahsi[5], the curve of the C_d covered the whole boundary layer of laminar, transition and turbulent flows, where defines the Reynolds number from laminar to transition regime as 'Begin Reynolds number, $Re_{tr,beg}$', and the Reynolds number turning into full turbulence regime defined as 'End Reynolds number, $Re_{tr,end}$', namely the beginning and ending positions of the viscous flow transition. The $Re_{tr,beg}$ and $Re_{tr,end}$ corresponding to different throat diameter were obtained in paper[9].

3. CFD simulation and verification

3.1 CFD modelling

Although many experimental data was being accumulated in support of the boundary layer transition theory, the performance of experimental apparatus was limited, and many coupling influence factors led to the difficulty. To easily analyse the transition processes of the boundary layer, it was necessary to perform the numerical simulation of the internal flow field of the sonic nozzle. It was usually used the model until the emergence of "Transition Shear-Stress Transport Model", which is good for the boundary layer transition with a high accuracy and wide application. The main idea is that in the near wall the model can be used to capture the viscous flow, and in the core flow region the model can avoid the disadvantage that the model is too sensitive to the inlet turbulent parameters.

3.2 Geometric model and Numerical scheme

The sonic nozzle structure was axisymmetric swirl strictly, which switches two-dimensional model into three-dimensional calculation. Thus, only a half of two-dimensional model can be used to realize the simulation of three-dimensional sonic nozzle model and the mesh number of the CFD model decreases immensely. The computing speed was enhanced greatly while the accuracy was not influenced. The geometry was established the corresponding topological structure to divide 'mesh point' and set the appropriate spacing and ratio to guarantee a sufficient structured mesh refinement near the wall, which has good convergence and high accuracy.

Three sonic nozzles with the throat diameter d of 1.919 mm, 3.808 mm, and 7.453 mm the inlet curvature radius of 2d and the diffuser angle of 4 degrees was chosen as an example. The total number of mesh was 200×500. Two-dimensional mesh generation adopted in the form of structured quadrilateral mesh, and it is perpendicular to the direction of flow to improve convergence. For this model, the position of x = 0 mm was the physical throat. An extensive of mesh testing was performed to guarantee a grid independence solution, and the residuals of all relevant parameters are less than 10^{-6} to guarantee the calculation accuracy. Density based solver with implicit formula was adopted for calculation, which is suitable for the high-speed compressible flow, and the second order upwind was applied for the finite volume discretization.

The fluid material was an ideal gas, and the operation pressure is zero, the inlet pressure was (0.1~90) MPa and the inlet stagnation temperature was fixed at 300 K. The inlet and outlet boundary conditions are pressure, and the back-pressure ratio was equal to or less than 0.1 to ensure the throat flow can reach critical conditions.

3.3 Comparison with experimental results

In order to verify the accuracy of the Transition SST model in the transition and turbulence regions with rough wall, the CFD results when Reynolds numbers ranges from 4.6×10^4 to 4.7×10^7 were compared with the experimental data by NIM. Figure 4 showed the experimental data by NIM and the simulation results at the corresponding relative equivalent sand roughness range. The symbol marked as '-Exp' represented the experimental results with $Ra/d = 8.4\times10^{-5}$, $Ra/d = 1.6\times10^{-4}$ and $Ra/d = 3.1\times10^{-4}$. According to Adams's study[11], the equivalent sand roughness K_s was about 5 times larger than the Ra , thus the $K_s/d = 4.2\times10^{-4}$, $K_s/d = 8.1\times10^{-4}$ and $K_s/d = 1.5\times10^{-3}$ i. While the '-CFD' represents the simulation data with K_s /d were equal to 1.3×10^{-5}, 1.3×10^{-4} and 1.3×10^{-3} respectively. As illustrated in the figure, the experimental data is in good accord with CFD results.

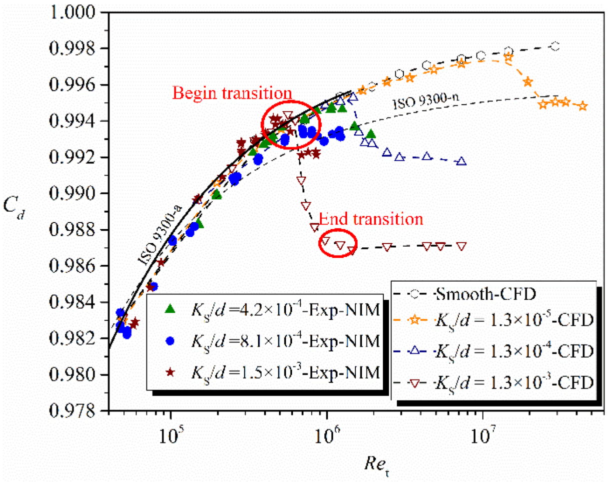

Figure 4: Comparisons of the experiment data with CFD results.

4. Results and discussion

4.1 Relationship between Ra, Rz, wa and K_s

In recent years, with the development of the measurement and evaluation technology of meso-micro surface structure, people began to apply this technology to the study of surface characteristics inner wall of pipe, and tried to establish the relationship between *Ra*, *Rz*, *wa* and K_s. The corresponding research results were mainly summarized into the following two types: 1) it is found that the K_s of the pipeline was expressed as the multiple of the average surface roughness *Ra* or the root mean square surface roughness *Rq*, based on the research of Adams[12], Allen[13] and langelandsvik[14]. 2) The K_s was expressed by the maximum surface roughness *Rz*, according to the research of Zheng[15] and Guo[16].

Based on the above research results, the different parameters of meso-micro surface structure for the 7.453 mm sonic nozzle *Ra, Rz,* and *wa* measured in section 2.1 were directly converted into K_s to simulate the variation law of C_d under different parameters, and compared with the experimental results, as shown in Figure 5. It can be seen from the Figure 5 that the *Rz* can be used as K_s to evaluate the C_d of the sonic nozzle, that is, $Rz \approx K_s$.

Therefore, it is more reasonable and effective with selecting the *Rz* as the K_s to evaluate the influence of meso-micro surface structure parameters on the C_d of sonic nozzle for ISO 9300.

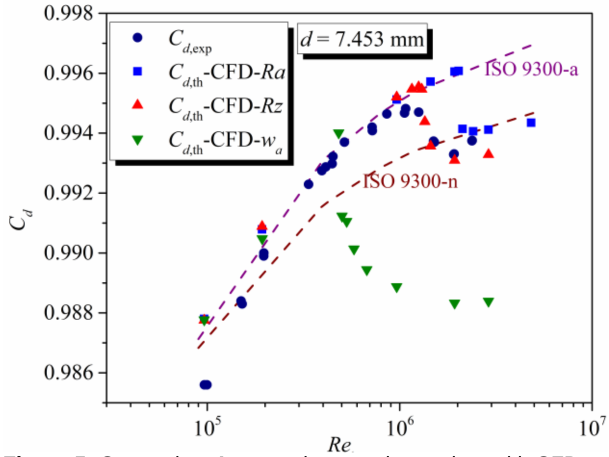

Figure 5: Comparison between the experiment data with CFD results of *Ra, Rz,* and *wa.*

4.2 Analysis characteristics of boundary layer transition

The comparison between CFD and experimental results of d = 7.453 mm sonic nozzle with the *Ra*, *Rz* and *wa* at Tu = 5% and 10% was made, as shown Figure 6. Figure 6 further showed that under the *Rz*, the CFD results of the C_d are in good agreement with the experimental results. It showed that the position of boundary layer transition was different, when the surface structure of sonic nozzle was different. With the increase of surface structure, the earlier the boundary layer transition position, the greater the decrease of C_d.

In addition, with the turbulence intensity at the inlet of sonic nozzle decreases, the position of the boundary layer transition will be delayed. The results showed that the effect of inlet turbulence intensity on the C_d should be paid more attention to in the future.

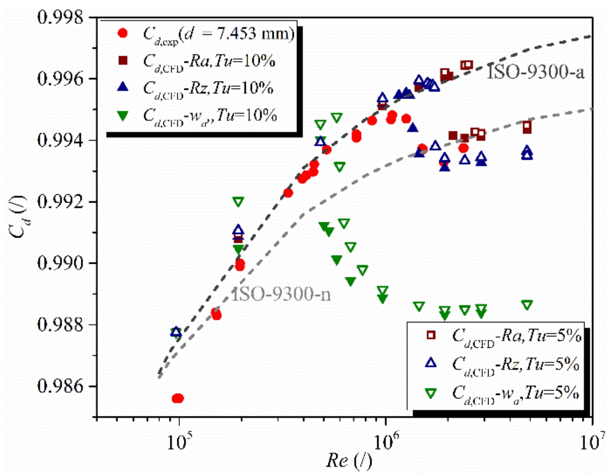

Figure 6: The CFD and experimental results of the C_d for d = 7.453 mm sonic nozzle with the *Ra*, *Rz* and *wa* at *Tu* =5% and 10%.

It can be seen from Figure 4 that the position of boundary layer transition was determined by the meso-micro surface structure parameters, resulting in instability of boundary layer transition. The flow and development of boundary layer were very sensitive to K_s. The *Rz* was taken as the K_s to deeply study the influence of surface structure parameters on the position of boundary layer transition for sonic nozzle.

A large number of simulation results were carried out for the sonic nozzle with throat diameter of 3.808 mm, and the values of $Re_{\mathrm{tr,beg}}$ and $Re_{\mathrm{tr,end}}$ were obtained under the conditions of different relative equivalent roughness K_s/d, as seen in paper[17]. It indicated that $Re_{\mathrm{tr,beg}}$ and $Re_{\mathrm{tr,end}}$ gradually decrease with the rise of the K_s/d.

Then, the changes of $Re_{\mathrm{tr,beg}}$ and K_s/d (or *Rz*/d) were shown in Figure 7, and their relationship was seen in Equation (1).

$$Re_{\mathrm{tr,beg}} = \frac{212}{Ks/d} = \frac{212}{Rz/d} \tag{1}$$

Based on Equation (1), it found that the $Re_{\mathrm{tr,beg}}$ is inversely proportional to the K_s/d or *Rz*/d. When the K_s/d or *Rz*/d of the sonic nozzle is known, the $Re_{\mathrm{tr,beg}}$ can be obtained, then the beginning position and the Reynolds number of the boundary layer transition can be known, and then the applicable range of the theoretical equation of the C_d can be defined.

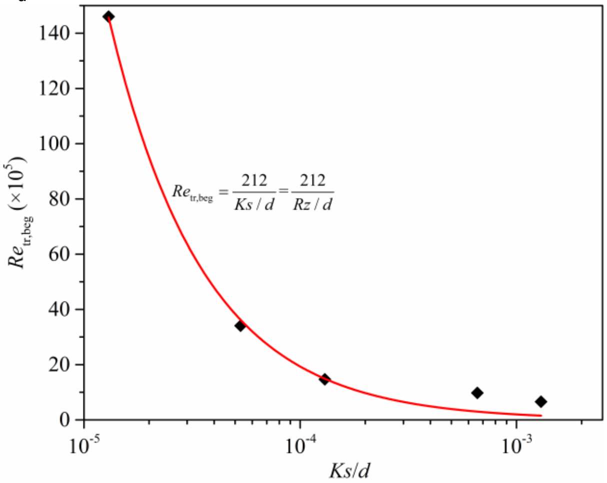

Figure 7: Relationship between K_s/d and $Re_{\mathrm{tr,beg}}$.

5. Conclusion

The meso-micro surface structure parameters of three sonic nozzles with throat diameters of 1.919 mm, 3.808 mm and 7.453mm were measured and analyzed by using a Taylor Hobson high-precision surface roughness meter, and their experimental C_d were calibrated on the pVTt facility in NIM. At the same time, a large number of simulations by the transition SST model were conducted to study the C_d of three sonic nozzles with different *Ra, Rz,* and *wa*. The effectiveness and accuracy of numerical model had been validated by the experimental data from NIM with the different K_s/d. The experimental results and the numerical results were as follows,

1) The position of boundary layer transition was different, when the surface structure of sonic nozzle was different. And, with the increase of surface structure, the earlier the position of boundary layer transition, the greater the decrease of C_d.
2) The CFD results of C_d were better consistent with the experimental results under the maximum roughness *Rz*. Therefore, it is more reasonable and effective to select the *Rz* when evaluating the influence of meso-micro surface parameters on the C_d of sonic nozzle. It was indicated that the *Rz* can be used as K_s to evaluate the C_d of the sonic nozzle, that is, $Rz \approx K_s$.
3) With the turbulence intensity at the inlet of sonic nozzle decreases, the position of the boundary layer transition will be delayed. The results showed that the effect of inlet turbulence intensity on the discharge coefficient should be paid more attention to in the future.
4) The inverse proportional relationship between the K_s/d (or *Rz*/d) of the sonic nozzle and the $Re_{\mathrm{tr,beg}}$ was established, which can specify the applicable range of the theoretical C_d, especially in the laminar boundary layer region.

The K_s/d or *Rz*/d effect on the C_d and $Re_{\mathrm{tr,beg}}$ are quite complex and should be investigated further. Besides, it will focus on the effect of *Tu* on the C_d in the future study.

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