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HISTORY AND PHILOSOPHY OF MEASUREMENT: A REALIST VIEW

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Abstract – According to the realist interpretation, measurement is the estimation of numerical relations (or ratios) between magnitudes of a quantitative attribute and a unit. The history of scientific measurement, from antiquity to the present may be interpreted as revealing a progressive deepening in the understanding of this position. First, the concept of ratio was broadened to include ratios between incommensurable magnitudes; second, the concept of a quantitative attribute was broadened to include non-extensive quantities; third, quantitative structure and its relations to ratios and real numbers were elaborated; and finally, the issue of empirically distinguishing between quantitative and non-quantitative structures was addressed. This interpretation of measurement understands it in a way that is continuous with scientific investigation in general, i.e., as an attempt to discover independently existing facts.

Keywords: realism, philosophy of measurement, history of measurement.

1. PHILOSOPHY OF MEASUREMENT: A REALIST VIEW

The realist philosophy of measurement differs from others (e.g., operationist and representationist philosophies) in three ways. First, it distinguishes **what is measured** from **how it is measured**. Second, it holds that what is measured are **attributes of things**, rather than **things themselves**. Third, it claims that in measurement, **numbers are discovered** rather than **assigned**.

The founder of operationism, P. W. Bridgman [1], said that the "concept of length involves as much as and nothing more than the set of operations by which length is determined" (p.5) viz., the operations used to measure length. That is, he thought that the operations used to measure something define the thing measured. This view exemplifies an enduring fallacy: the failure to distinguish a relation from the terms it holds between. Bridgman thought that just because an attribute like length is only **known through measurement**, it must only **exist through measurement**, it being, he thought vain to distinguish nature as it is from nature as it is known. The fallacy here is that unless nature exists independently of being known, there would be nothing to know. Likewise, unless the things we measure exist independently of being measured, there would be nothing there to measure. Measurement procedures are a means of attempting to find something out. They cannot at the same time constitute what it is that is found out.

According to S. S. Stevens [2], measurement is "the assignment of numerals to **objects or events** according to rule" (p.667; my emphasis). In stressing objects and events, Stevens, like Bridgman before him, wanted to avoid reference to the **attributes** of things (their **properties** and **relations**). However, the fact that different kinds of measures can be made of the same object (say, measures of mass and volume) means that measurements are not individuated by the objects measured. The rejection of operationism means that they cannot be individuated by the rules or measurement operations involved. All that is left to individuate them is attributes. That is, what makes a measurement of, say, mass, different from one of, say, volume, must be the **kinds of attributes** identified (i.e., the mass of the object as opposed to its volume, where each is understood as a property).

Bertrand Russell [3] said that measurement is "the correlation, with numbers, of entities which are **not numbers**" (p.158; my emphasis). This is the representational view and it is this view of measurement that has received the majority of attention over the past century. According to this view, measurement depends upon an isomorphism between an empirical system and a numerical system. However, if an empirical system is isomorphic to a numerical system and numerical systems are characterised by structure alone, then such empirical systems must **instantiate** the relevant numerical system. Thus, it follows that in measurement numbers are not correlated with anything and certainly not correlated with things that are not numbers. Rather, **numerical relations between attributes are discovered**.

Furthermore, any specification of the structure of quantitative attributes that is rich enough to accommodate the attributes measured in physics entails that ratios of magnitudes of a quantitative attribute have the structure of the positive real numbers. According to the realist view, such ratios **are** the real numbers. Taking this philosophical position seriously provides a particular specification of what measurement is: **measurement is the attempt to estimate the ratio between two instances of a quantitative attribute, the first being the magnitude measured, and the second being the unit employed**. According to the realist view, one important aim of measurement is to identify the first instance (the measured attribute) through attempting to estimate its relation (or ratio) with the second instance (the unit). From this standpoint, the practice of measurement is continuous with scientific investigation generally: it is the attempt *to find out* something. This implies that measurement is continuous with scientific investigation in another way as well: the issue of whether an attribute is measurable turns on the empirical issue of whether it possesses quantitative structure. This means that measurement is not a completely general scientific method, one suited to every scientific question. Instead, measurement is a method tuned to specific scientific questions: those involving attributes possessing quantitative structure.

This standpoint provides a perspective on the history of scientific measurement that displays how the understanding of measurement has developed to meet challenges within science's history. In the remainder of this paper, this historical claim is illustrated through four episodes: (1) the generalisation in antiquity of the ancient concept of **measure** to that of **ratio**; (2) the generalisation in the middle ages of the concept of quantity to include non-extensive (i.e., **intensive**) **magnitudes**; (3) the axiomatisation of the concept of **unbounded, continuous quantity** at the beginning of the twentieth century; and (4) the specification later in that century of empirical tests capable of distinguishing **quantitative** from **non-quantitative** structures.

2. HISTORY OF MEASUREMENT: A REALIST VIEW

2.1 *The Generalization of Measure to Ratio.*

The first systematic contribution to the philosophy of measurement available to us in the historical record is Book V of Euclid's *Elements* [4]. It involves a detailed treatment of ratios[1]. Ratios are considered by first introducing another relation, viz., that of **measure**. The **measure of one magnitude, *b*, relative to another, *a*,** is the whole number, ***n***, such that ***b = na***. If magnitude *a* is known and it is also known what a multiple of *a* is, then knowing that $b = na$ (where n is a whole number) identifies b explicitly as a quantitative composite of something known. While this relation holds between certain pairs of magnitudes of the same general kind, it was known not to hold between other pairs, such as the lengths of the side and diagonal of a square. In this sense of **measure**, the set of all lengths, for example, lacks a common measure. Book V remedies this by generalizing the concept of **measure** to that of **ratio**.

That the concept of **ratio** is a generalisation of **measure** can be seen if for these two relations the conditions for **sameness of relation** are set out in parallel. Let a, b, c and d be any magnitudes of the same general kind (say, any specific lengths). The **measure** of a relative to b **is the same as** the **measure** of c relative to d if and only if for any natural numbers n,

(i) $a = nb$ if and only if $c = nd$.

The **ratio** of a relative to b **is the same as** the **ratio** of c relative to d if and only if for any natural numbers m and n,

(ii) $ma < nb$ if and only if $mc < nd$;
(iii) $ma = nb$ if and only if $mc = nd$; and
(iv) $ma > nb$ if and only if $mc > nd$.

Measure is the special case of **ratio** when $m = 1$ and for some n, $ma = nb$.

The importance of *sameness of ratio* for understanding measurement is that it means that a is quantitatively related to b even when there are no natural numbers, m and n, such that $ma = nb$ (i.e., when a and b are incommensurable). For even then, for any pair of natural numbers, m and n, if $ma > nb$, the magnitude of a relative to b exceeds the numerical ratio, n/m, and if $ma < nb$, then it is exceeded by the numerical relation n/m and, so, the magnitude of a relative to b is uniquely located within the ordered sequence of all numerical ratios, even though it does not equal any such ratio. In modern terms, measuring a in units of b estimates this location [6], that is, estimates the positive real number r, such that $a = rb$.

That this concept of ratio resembles the modern concept of real number has not escaped attention [7-9]. When it is realized that the Euclidean concept of number ('A number is a multitude composed of units' Definition 2, Book VII[2]) corresponds to the ancient concept of measure and that the concept of ratio generalizes the concept of measure in a way similar to that in which the modern concept of real number generalises that of natural number, it can be seen that the concept of ratio and that of real number are practically the same concept[3].

That ratios are instances of real numbers is the realist view. This view has a number of advantages over alternatives. First, it identifies numbers as **relations** of a definite kind (i.e., as ratios) and, so, means that no special ontological category is needed to accommodate them. Second, it locates numbers within the **spatiotemporal world of experience** and, so does not require a special realm of being (say, a realm of abstract entities) outside of space and time for numbers to inhabit. Third, if numbers are spatiotemporally located relations, then no special problem attaches to explaining **our knowledge of them**. We know about numbers in the same way that we know about anything, that is, via sensory experience and logic. Third, if numbers are ratios of magnitudes of quantitative attributes, then there is no mystery in understanding their **application in measurement**. Numbers and

[1] The thirteen books of the *Elements* are said [5] to have been compiled by Euclid during the fourth century BC and Book V is attributed to Eudoxos of Cnidus (408-355 BC).

[2] ([4] p. 277).

[3] This is how Frege [10] and Whitehead & Russell [11] understood the real numbers, and it is the view revived by Forrest and Armstrong [12] and Michell [13].

truths about numbers are applicable to the empirical context of measurement because numbers, as ratios, are instantiated within that same context.

Because of their special concept of number, the ancient Greeks could not interpret ratios as of a kind with numbers. However, from a practical point of view, the concept of ratio expounded in Book V was sufficient to provide an understanding of the place of numbers in the practice of measurement; of the sense in which incommensurable magnitudes are measurable; and what it is that is estimated in measurement. This is why this concept was a source of inspiration for subsequent quantitative scientists, especially those of the fourteenth[4] and seventeenth centuries[5].

To understand the definition of **ratio** given in *The Elements*, viz., that 'a **ratio** is a sort of relation in respect of size between two magnitudes of the same kind'[6], it is necessary to know what magnitudes of the same kind are. This requires considering the concept of a quantitative attribute in a general way, i.e., in abstraction from the objects to which instances of the attribute are necessarily tied.

2.1 *Generalisation of Quantity.*

Given a quantitative attribute relative to which multiples are identifiable for some range (as, for example, humanly manageable multiples of lengths are identifiable), the concept of ratio is interpretable. In the ancient world, this was so for the geometric attributes and others, such as time. Furthermore, measurement is such an elegant concept that even with attributes apparently lacking multiples but capable of increase or decrease (like temperature, for example), the temptation to treat them as quantitative seems irresistible.

For example, Aristotle thought of temperature as a quality[7], not a quantity and in his metaphysics the categories of quantity and quality were thought of as distinct. Yet he wrote of increase and decrease in temperature in quantitative terms[8]: conceptualising them in relation to the proportion of hot or cold parts constituting the object involved. While this might explain the fact that mixing volumes of hot and cold water results in a liquid of intermediate temperature, it does not explain how a liquid could be homogeneously tepid, as might result, say, when a cold liquid is heated.

When an attribute of an object, such as its length, increases, a new part possessing the extra length is added to the object involved. This is *extensive addition*. However, in cases of increase in temperature, addition in this extensive sense does not occur. When liquids of different temperatures are combined, as the medieval philosopher, John Duns Scotus, noted, the volumes add extensively but not the temperatures:

> Sometimes a tepid [degree] added to a tepid [degree] in diverse subjects does not increase [heat]; [but] this is accidental, on account of the extension and dispersal of the parts. If [the new tepid degree] were in the same [extended] part of a subject with the pre-existing tepidness, then [the subject] would certainly be increased, and be hotter.[9]

That is, in the case of qualitative increase[10], the possibility needs to be considered that different degrees of the quality can be added together in ways other than via addition of spatially extended parts possessing those degrees. This requires thinking of a causal process that increases the degree of the qualitative attribute (say, adds to the temperature of a liquid) without adding spatially extended parts to the object possessing that attribute (in this case, the liquid).

Scotus's insight was a major conceptual breakthrough, although one still largely unsung. He saw that the fact that an attribute is quantitative concerns its internal structure, while the fact that it is extensive has to do with its external relations, that is, with how humans interact with it. Had we been constructed differently, with radically different sensory-motor capacities and forced into a very different range of causal interactions with physical objects, the geometric attributes may well have been experienced by us as *intensive*[11], rather than as extensive attributes[12]. Scotus' insight is part of the process of removing features relative to the human observer from the understanding of the phenomena under investigation.

This is an inevitable consequence of taking a realist approach to science, one that distinguishes **the phenomena under investigation** from **the investigation of the phenomena**. In measurement, the phenomena under investigation involve quantitative attributes. Until the scientific revolution, the only way that was known whereby the specifically quantitative

[4] See Molland [14] for a comprehensive examination of the concept of ratio in fourteenth century science.

[5] Galileo abandoned his medical studies upon hearing an exposition of Book V of *The Elements* and was writing a commentary on it upon his deathbed [15]; and '… within the area of inquiry which Galileo deals with mathematically, only a single theory is rigorously applied. This is the theory of proportionality of general magnitudes developed by Eudoxos and found in the Fifth Book of Euclid's *Elements*' ([16] p.236).

[6] [4] (p.114).

[7] See the discussion of the 'elementary qualities' of *hot* and *cold* in Bk. II, Ch. 2 of De *Generatione et Corruptione* [17].

[8] See Bk. II, Ch. 7, 334^b8-30, *De Generatione et Corruptione* [17] (pp.521-522).

[9] As quoted in [18] (p.190).

[10] In the Middle Ages, the degrees of a quality, such as temperature, were known as the 'latitudes of a form.' *Latitudo* was the Latin translation of the Greek, *πλατος* (*platos*), which 'was connected with the terms for tightening and relaxing the strings of a lyre' [19] (p.57). The length of a lyre's string was analogous to the extent of a quality and its tightness to its intensity.

[11] In the Middle Ages, increase and decrease of a quality was called the 'intension and remission of forms' and, so, the qualities themselves became known as 'intensive magnitudes.'

[12] See [20].

features of such attributes could be investigated was via extensive addition. In order to open up the possibility of including a wider range of attributes in quantitative science, it was necessary to decouple the concept of quantity from that of extensive addition. This was the achievement of Duns Scotus and the scientists who followed him.

Medieval scientists recognised that extensive and intensive attributes are, alike, quantitative. This is evident in Nicole Oresme's observation that intensive quantities may be imagined by analogy with length:

> every intensity which can be acquired successively ought to be imagined by a straight line … For whatever ratio is found to exist between intensity and intensity, in relating intensities of the same kind, a similar ratio is found to exist between line and line and vice versa … Therefore, the measure of intensities can be fittingly imagined as the measure of lines.[13]

This way of thinking about quantitative attributes abstracts them from the specific contexts of their occurrence. As a result, Scotus' breakthrough created an intellectual milieu in which attributes, hitherto thought of as qualities, could be hypothesised as quantitative without requiring operations of extensive addition. This way of thinking flourished in the fourteenth century [22] and, in turn, created a climate of thought sustaining such Pythagorean sentiments as Galileo's conviction that 'The book of nature is written in mathematical language'[14] and the expansion of measurement practices that accompanied the scientific revolution of the seventeenth century [24].

2.2 Unbounded, Continuous Quantity

Scotus' insight meant that the fact that some attribute does not, in our experience, sustain extensive addition is no impediment to entertaining the hypothesis that the attribute is quantitative. Considering such a hypothesis raises two further questions. First, what is the character of quantitative structure? There is no point debating whether some attribute is quantitative, if the character of quantitative structure is not understood. Second, what kinds of evidence distinguish quantitative from non-quantitative structure? If the issue of whether some attribute is quantitative is to be tested, some idea is required of what evidence is relevant to this question.

The first question was answered by the mathematician, Otto Hölder [25][15]. Hölder articulated conditions characterising the structure of unbounded, continuous quantitative attributes, this structure being the paradigm for the kinds of attributes measured in physics. Call any such attribute Q and let its different levels (the specific magnitudes of Q) be designated by $a, b, c, \ldots$. For any three levels, a, b, and c, of Q, let $a + b = c$ if and only if c is entirely composed of discrete parts a and b. According to Hölder, the structure of such an attribute is characterised by seven conditions:

1. Given any two magnitudes, a and b, of Q, one and only one of the following is true:
   (i) a is identical to b (i.e., $a = b$ and $b = a$);
   (ii) a is greater than b and b is less than a (i.e., $a > b$ & $b < a$); or
   (iii) b is greater than a and a is less than b (i.e., $b > a$ & $a < b$).
2. For every magnitude, a, of Q, there exists a b in Q such that $b < a$.
3. For every pair of magnitudes, a and b, in Q, there exists a magnitude, c, in Q such that $a + b = c$.
4. For every pair of magnitudes, a and b, in Q, $a + b > a$ and $a + b > b$.
5. For every pair of magnitudes, a and b, in Q, if $a < b$, then there exists magnitudes, c and d, in Q such that $a + c = b$ and $d + a = b$.
6. For every triple of magnitudes, a, b, and c, in Q, $(a + b) + c = a + (b + c)$.
7. For every pair of classes, ϕ and ψ, of magnitudes of Q, such that
   (i) each magnitude of Q belongs to one and only one of ϕ and ψ;
   (ii) neither ϕ nor ψ is empty; and
   (iii) every magnitude in ϕ is less than each magnitude in ψ,
   there exists a magnitude x in Q such that for every other magnitude, x', in Q, if $x' < x$, then $x' \in \phi$ and if $x' > x$, then $x' \in \psi$ (depending on the particular case, x may belong to either class).

Hölder proved that if an attribute has this structure, then the system of ratios of its magnitudes is isomorphic to the system of positive real numbers and he went on to show that with any magnitude as the unit, each magnitude is measured by a real number. This made the link between the concepts of quantitative structure and ratio (in the sense of Euclid's *Elements*) explicit.

From the realist perspective, the significance of Hölder's achievement cannot be underestimated. It brings into the open the character of quantitative structure. This means that when a scientist proposes that some hitherto unmeasured attribute is measurable, as for example psychologists did about a century ago, it is clear what is being proposed. It is then up to the scientists involved to put theoretical flesh onto the skeletal structure provided by Hölder. Only when this is done is a scientific theory actually proposed.

In any area of investigation, a scientist is always free to hypothesise that an attribute is quantitative. However, there is no *a priori* reason to suppose that just any attribute will possess quantitative structure. Thus, scientific caution requires that the hypothesis of quantitative structure always be evaluated relative to

[13] *De Configurationibus I, i,* [21] (pp.165-167).
[14] *Il Saggiatore,* [23] (p.237).
[15] [26 & 27] provide an English translation of Hölder.

available evidence. This raises the issue of the kind of evidence that is relevant.

Inspection of Hölder's conditions reveals that even in the case of extensive attributes these conditions are not all directly testable. Some (e.g., condition 6) are; but others (e.g., 2 and 7) are not. How these conditions might be tested for non-extensive attributes becomes clear when scientists begin to think about the kinds of evidence that might tell against them. This was not a major issue in physical science, in part, because physicists restricted themselves to measuring attributes that were either extensive or functionally related to attributes already accepted as measurable; and in part because the Pythagorean vision of reality (viz., that all physical attributes must be quantitative) informed much of the development of physical science. It was the proposed extension of measurement to non-physical science that made the task of distinguishing quantitative from non-quantitative structure appear to be urgent[16].

2.3 *Quantitative versus Non-Quantitative Structure*

The scientist, Hermann von Helmholtz, was one of the first to consider kinds of evidence for testing the hypothesis that attributes are quantitative[17]. He asked: 'what is the objective sense of our expressing relationships between real objects as magnitudes, by using denominate numbers; and under what conditions can we do this?'[18] He distinguished 'additive' from 'non-additive' magnitudes, the former being those, such as mass, time, and the geometric attributes, relative to which operations of extensive addition are identifiable, and the latter those where quantitative structure is identified via constants in numerical laws, as, for example, density is a constant ratio of mass to volume for each different kind of substance. This was the distinction that N. R. Campbell [34] popularised as 'fundamental' and 'derived' measurement. From the realist viewpoint, neither pair of terms is optimal. The distinction is not really between kinds of magnitudes, nor kinds of measurements. It is one between the character of evidence for quantitative structure, viz., whether it is **direct** or **indirect**.

In the case of direct evidence, an operation of concatenation is found via which distinct magnitudes of the attribute can be added extensively. For example, such an operation is easily identified in the case of weight when objects are placed on the same pan of a beam balance. Such an operation allows a direct test of the associative and commutative properties of addition. Of course, one can never directly test any of these axioms across the full range of the attribute, or all of the axioms even across a limited range.

From the realist perspective, there will be, in principle, indefinitely many different ways in which indirect evidence may be obtained for the hypothesis that some attribute is quantitative. This is because realists take nature to be infinitely complex. However, with respect to any proposed way of gaining such evidence, the burden of proof is always on those advocating it to show that it is capable of distinguishing quantitative from non-quantitative structure.

The logic of one way of gaining indirect evidence became clear when the psychologist, R. D. Luce and associates, expounded the theory of 'simultaneous conjoint measurement'[19]. Again, from the realist point of view, this term is suboptimal because what they were theorising about was not so much a method of measurement as a context within which indirect evidence for quantitative structure could be collected simultaneously for a number of attributes. Their proposal also reveals the logic of Helmholtz's 'non-additive' magnitudes and Campbell's 'derived' measurement.

At the most general level, this theory is about the way in which merely ordinal structure on a product set, $Y \times Z$, is diagnostic of quantitative structure intrinsic to Y and Z. The theory proves that order upon $Y \times Z$ satisfies certain specific conditions (e.g., a potentially infinite hierarchy of *cancellation conditions*[20]) if and only if Y and Z are quantitative in structure and $g(Y \times Z) = f(Y, Z)$, where g and f are real valued functions and f reduces to additivity under some monotonic transformation. Importantly for the understanding of measurement, the theory applies in contexts where one attribute, X, is a non-interactive, monomial function of two other attributes, Y and Z, (e.g., when $X = Y^p.Z^q$ (where p and q are positive or negative integers)) because then X maps onto $Y \times Z$. Instances of what Campbell called derived measurement are of this form (e.g., $density = mass^1.volume^{-1}$). This means that the way in which attributes quantified via derived measurement possess quantitative structure is displayed via conjoint measurement[21]. Conjoint measurement also

[16] It was Fechner's psychophysics [28] that elicited the strongest opposition [29].

[17] [30]. According to Heidelberger [29 & 31], Helmholtz was critical of Fechner's psychophysics, although that is not clear from Helmholtz's seminal paper [32].

[18] The quote is from an English translation of [32], viz., [33] (p.75).

[19] The definitive paper is [35], although [36] gives the most complete exposition of the theory of conjoint measurement. Regarding the issue of empirical tests of quantitative structure via conjoint measurement, [37] is as important as [35].

[20] The best known cancellation conditions are **single cancellation** (sometimes called 'independence') and **double cancellation**. An order, $\leq$, upon a product set $Y \times Z$ satisfies **single cancellation** if and only if (i) for all levels a and b in Y and any level x in Z, if $ax \leq bx$, then for all levels y of Z, $ay \leq by$, (where ax, etc. are elements of $Y \times Z$) and (ii) for any level a in Y and all levels x and y in Z, if $ax \leq ay$, then for all levels b in Y, $bx \leq by$; and satisfies **double cancellation** if and only if for every three levels, a, b, and c, of Y and every three levels, x, y, and z, of Z, if $bx \leq ay$ and $cy \leq bz$, then $cx \leq az$.

[21] Some philosophers, such as Carnap [38], see the relationship between derived measures and attributes already meas-

has implications for the measurement of non-physical attributes, such as psychological ones. Psychological attributes are experienced directly only as ordinal, at best, and yet within psychology they are routinely presumed to be quantitative and measurable. The theory of conjoint measurement makes the distinction between mere order and quantity explicit.

This theory shows that the question of whether an attribute is quantitative is the same as any other scientific question in the sense that it is open to empirical investigation and refutation. From the realist standpoint, this is the way things have to be. Hölder's characterisation of quantitative structure shows it to be a specific empirical condition in the sense that there is no logical necessity that the conditions he specifies should obtain with regard to any attribute at all. The theory of conjoint measurement brings out explicitly the kind of difference the hypothesis of quantitative structure makes.

3. PHILOSOPHICAL RESUMÉ

Measurement is a process whereby the numerical relation between a magnitude of a quantitative attribute and a unit of the same attribute is estimated using some procedure, often a standardised set of operations. The realist view of measurement is distinguished from other perspectives by the fact that all elements of the process are given a naturalistic interpretation and, in particular, both the **attribute** and the **numerical relation** are taken to be real features of the empirical context of measurement. However, neither of these (i.e., the realist interpretation of attributes and of numbers) is peculiar to the realist view of measurement. Some representationists (e.g., Campbell [34]) understood measurement as involving attributes and interpreted numbers realistically (Campbell thought of natural numbers as properties of aggregates). The most distinctive feature of the realist view is that according to it measurement is continuous with science generally and involves nothing that is logically special or peculiar to it alone.

The idea that measurement is somehow philosophically different to other aspects of the scientific enterprise is still prevalent. For example, the operationist view that other concepts of science are **defined by measurement operations** assigns to measurement procedures a logical priority over other scientific concepts and raises the question of how the procedures of measurement are themselves to be defined[22]; and the representational view, that measurement involves a distinctive relation, that of **numerical representation**, makes measurement appear a strange hybrid of the empirical and the conventional and raises the question of the apparently 'unreasonable effectiveness of mathematics in the natural sciences' [40][23]. From the realist perspective, measurement is continuous with other features of science.

Measurement can only be based upon the fact that the relevant attribute possesses quantitative structure. Non-quantitative attributes cannot be measured (which is not to say that they cannot be studied scientifically). If an attribute is quantitative, then measurement is a possibility. No attribute can be presumed quantitative and the scientific assessment that an attribute is quantitative must be based upon relevant evidence. The hypothesis that some attribute is quantitative, like any in science, says that specific empirical conditions obtain (e.g., those articulated by Hölder [25]) and it, thereby, rules out other possibilities. The general, scientific method of critical inquiry, according to which hypotheses are only accepted in science following the success of serious attempts to test them, applies to this hypothesis as much as to any.

When the hypothesis that an attribute is quantitative is accepted, then along with that hypothesis, as part of the same theoretical package, it is accepted that different magnitudes of the attribute stand in relations of ratio, these relations being instances of real numbers. Thus, according to the realist view, real numbers are taken to be located in the empirical context of measurement as intrinsic features of that context. When scientists set about devising practical, standardised procedures for measuring, it is precisely these real numbers (ratios between unknown magnitudes and the unit adopted) that the scientists are attempting to identify. There is nothing in any of this that makes measurement different in principle to anything else that is done in science. It is, like so many methods used in science, a method for trying to find something out. The most important factor distinguishing measurement from other methods is the context of its application: its context is that of quantitative attributes and the ratios that they sustain.

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ured (such as that between density and mass and volume) as definitional. Application of the theory of conjoint measurement to derived measurement shows that these relationships are not definitional: they are testable, empirical hypotheses (see [36]).

[22] See the critique of operationism in [39].

[23] See the critique of representationism in [41-42].

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