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Processing of shaking table test data of a historic masonry structure by graph-based methods

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***Abstract* – A mockup representing a historic masonry vault was tested on the shaking table and finally brought to collapse. During the seismic test, a video was taken and then processed by the motion magnification algorithm to magnify the smallest displacements. Subsequently, each frame of the video was translated into a graph to be analyzed by the graph's centrality measures. Some of these parameter values significantly changed along the video before the collapse. Therefore, such parameters showed the potential to predict the collapse of the structure.**

I. INTRODUCTION

The instrumental monitoring of structures plays a fundamental role in detecting damage processes in ancient and modern buildings. However, conventional methods typically provide information about the *current* status of the structures. Engineers are able to infer the structural health (SH) and the state of integrity of the buildings based on simulations and analyses of numerical models, but this requires experience and in-depth knowledge of the structure. Moreover, most methods need continuous or periodic monitoring of the structure. Therefore, there is a strong boost towards automatic or semi-automatic SH systems. In this paper, we explore the potentiality of a new method to outline a collapse alert for historic masonry structures. The system is based on methodologies and concepts taken from the graph theory (GT), topology analysis (TA), motion magnification (MM). Motion magnification [1] acts as a microscope for tiny movements present in common digital videos. Its advantages in the study of ancient monuments are documented in [2, 3, 4, 5]. A graph is a mathematical object composed of nodes and links (or edges) connecting the nodes expressing some type of inter-relations, see the very good introductions of [6, 7]. The topology considers continuous deformations (homeomorphisms) of an object from a geometrical point of view [8, 9]. If a property of the object is invariant with respect to the deformations, it is called a topological invariant. Here we consider only some very basic results, since the mathematical treatment is very technical and beyond the scope of the paper. The proposed method was experimented by the shaking table test of a mockup of a historic masonry structure. The structure was eventually brought to collapse by seismic excitations reproducing a natural earthquake of weak intensity. During the seismic test a video was taken, then, the video was magnified by the phase-based MM algorithm [1] in order to amplify the deformations of the structure. At this point, each frame of the video was transformed into a graph, allowing the use of the tools of GT and TA. Some of these tools may provide invariants that should remain quite uniform along the video frames, unless a catastrophic event occurs. Of course, a collapse of the structure would be clear also to the naked eye, but some seconds before it could be not so evident. In fact, the structure undergoes deformations that may still be not destructive, although very dangerous. Anomalies in the structure may be sensed and even detected by accelerometers or more sophisticated devices [10]. Nevertheless, the overall picture remains difficult to interpret quickly and needs time consuming data processing and analyses. So we explored the possibility to set an early alert indicating that structural deformations became so large to significantly change the topological geometry of the structure. For example, let us consider a structure and suppose that thousands of vibration sensors (e.g. accelerometers, displacement sensors, etc.) are located regularly distributed on the structure. Using such data, a detailed finite element description (discrete crack model,) can be created to well represent the dynamic failure of the structure, as long as enough computing power is provided. However, at least in principle, immediately before the collapse the structure stiffness is lost and the model is not reliable anymore.

Thus, even in the most favorable conditions, near the collapse inferences about the building structural health are very difficult. We tried to face this issue taking advantage of Graph theory that facilitates the treatment of high dimensional problems reducing it to a lower dimensionality and providing topological measurements.

II. METHODS

The Graph theory is an established branch of mathematics that offers a number of useful descriptive parameters [7, 8]. Usually, if the graph is derived from technological processes or natural phenomena is called a network. A graph G is a collection of relations among n objects called nodes or vertices, linked by edges, indicating that some sort of interaction exists between a pair of nodes. A symmetric matrix called adjacency matrix **A** represents the interactions: a_{ij} entries are 1 if an edge links node *i* to node *j*, 0 otherwise. A graph is connected if a path of edges exists from any node to any other node: here we consider only the connected component of G, discarding non-connected nodes from the analysis. The graph properties are described by several parameters such as Closeness, Betweenness, Degree and many others, usually involving heavy computations if the graph is large. We use some of them to quantify the structural health of the building. Actually there are others centrality measures that could produce good alert signals, but one has to assess also the calculation time, the specificity, false alarm rate, the cost-effectiveness, to choice the most suitable.

MM acts like a microscope for micro-motions in digital videos unveiling visual patterns hardly visible to the naked eye, but saving the topology of the image sequence.

The MM algorithm works on the pixel intensity value along all the video frames, forming a time-series. Measurements with conventional velocimeters and accelerometers are surely more precise and accurate, but are also expensive and much less practical [10]. Moreover, since each pixel produces a time-series, a huge number of contactless "virtual sensors" are made available. This is important, as it is well known that video processing takes a lot of time and resources, preventing a viable application to the real world. Of course, this is a crucial issue for every real-time alert system. Topology offers the basis of theoretical support to link the GT to the motion magnification. The point is to transfer the topology of the magnified frames into the topology of graphs. The MM algorithm is considered as a continuous deformation that keeps the topological properties of the objects, allowing the subsequent application of the graph theory. Until these topological properties hold, GT allows the calculation of the graph parameters.

We will not analyze in details this aspect. We only note that the approach is different from the topological data analysis (TDA) described in [11] and also from Graph Signal Processing (GSP) [8], although there is a common ground. Moreover, to the best of our knowledge, the first *clear* proposal to use the GSP or the graph eigenvalues in the vibration analysis is in [12, 13]. However, [12] uses the GSP and MEMS sensors (but not the Fiedler eigenvalue) to study a steel beam, while [13] uses the Fiedler eigenvalue, but only for an industrial machinery. Our approach is radically different, since we use the pixels of a video as "virtual sensors" [3, 4, 5, 14] and the Fiedler eigenvalue to monitor a real masonry structure during a seismic test. Moreover, our procedure is much simpler and intuitive through the use of the motion magnification. Therefore, this could be the first time that the spectral graph parameters are applied to the analysis of a masonry structure.

A. Graph parameters

As said before, graph properties are described by several parameters. All these techniques unfortunately involve heavy computations. A short list of the most common parameters is described below.

- *Closeness*

It is the inverse of centrality associated with a node. The sum of the shortest path lengths between a given node and all other nodes in the graph. Vertices that tend to have short geodesic distances to other vertices in the graph have higher closeness.

- *Betweenness*

This is the total number of shortest paths between every possible pair of nodes that pass through the given node. Vertices that occur on many shortest paths between other vertices have higher Betweenness.

- *Degree*

In this context the Degree is the number of edges from a node. Intuitively, a high degree is quite often associated to a fundamental role for the node.

Now, remembering that the eigenvalues of a graph G are defined as:

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \tag{1}$$

where **A** is the *n*x*n* adjacency matrix of G, it is possible to calculate the so-called eigenvalue spectrum:

$$0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \tag{2}$$

A similar spectrum is calculated from the Laplacian matrix **L**, defined as **L** = **D** – **A**, where **D** is the node degree matrix. The eigenvalues of **A** or **L** are versatile tools, because they can represent the majority of a graph characteristics. In particular, the second eigenvalue of the Laplacian in ascending order λ_2, is called algebraic connectivity or Fiedler eigenvalue. It is greater than zero if the graph is connected: the larger the algebraic connectivity, the better the connectivity, the robustness and the resilience capability of a network [6, 7], and in our case, of the structure. Actually, we take the inverse of the algebraic connectivity, as we are interested in the level of the network deconstruction. Another important eigenvalue parameter is λ_n from the adjacency matrix **A**.

Our approach is to build for every k^{th} frame of the magnified video (before the collapse), a graph G_k and the matrices $\mathbf{A}_k$ and $\mathbf{L}_k$, in order to calculate their spectra (2). In particular, we are interested in λ_n^k and λ_2^k, for $k = 1,\ldots N_{frame}$.

The Fiedler eigenvalue, the maximum eigenvalue of **A** and the spectral parameters listed below may be considered as potential alert signals.

- *Dynamical Importance*

The Dynamical Importance is variation of the max eigenvalue after a node has been removed. Indicates how much the node is influential with respect to the others.

- *Estrada Index*

It represents the sum of closed walks of different lengths in the network starting and ending at a given vertex.

- *K-core*

Every subgraph has a vertex of Degree at most K. That is, some vertices in the subgraph touches K or fewer of the subgraph's edges. Nodes are ranked accordingly.

- *AV11 Index*

For a given graph G, the AV11 index identifies simultaneously the K most "important" nodes using the graph eigenvalues, as defined in [16].

- *Other parameters*

There is also a group of parameters (e.g. Tenacity, Integrity, Scattering, Toughness etc.) specifically developed to quantify the robustness of a graph, defined as the capacity to preserve the connectivity after some nodes or edges are deleted. But their calculation is "nondeterministic polynomial-time complete" (NP-complete), therefore they were not taken into consideration [17] in the present work.

B. Motion magnification

Vibration monitoring of structures is a major issue for the damage detection. Today, a new digital image processing method, namely the motion magnification, allows to magnify small displacements in video motions. Motion magnification acts like a microscope for motion in video sequences, but affecting only some groups of pixels, unveiling motions hardly visible with the naked eye. MM uses the spatial resolution of the video-camera to extract physical properties from images to make inferences about the dynamical behavior of the object. Researchers are interested in assessing the method's feasibility, since conventional devices are surely more precise, but expensive and much less practical. A number of experiments conducted on simple geometries like rods and other small objects as well as on bridges, have demonstrated the reliability of this methodology compared to contact accelerometers and laser vibrometers. We have extend the MM to the indoor analysis of historical mockups, and, generally speaking to the cultural heritage protection. Results show that MM analysis allows a visual identification of vibration mode shapes and of the most vulnerable elements of the structures. Though our equipment was of low quality in order to test the methodology in an adverse environment, results were very good. Evaluating the health of large structures in a short time span and possibly by simple devices that do not require expert operators, may be a pivotal issue in civil engineering. Thus, the availability of intuitive methodologies such as those based on a digital acquisition of images may result in a major breakthrough. The analysis of image sequences in the field of civil engineering is not new. For many years attempts to produce qualitative (visual) and even quantitative analysis using high quality videos of large structures have been conducted, but with poor results. This because of the resolution in terms of pixels, of the noise, of the camera frame rate, computer time and finally because of the lack of appropriate algorithms able to deal with the extremely small motions related to a building displacement. These and others limitations have restricted in the past the applications of digital vision methodologies to just a few cases. Nevertheless, recently important advances have been obtained by MIT [1]. Their algorithm, named motion magnification, seems able to act like a microscope for motion and, more importantly, in a reasonably short elaboration time. The latter point is crucial, as it is well known that image processing takes a lot of time and resources. Therefore, any viable approach must consider the reduction of the calculation time as an absolute priority. The basic MM version looks at intensity variations of each pixel, revealing small motions linearly related to intensity changes through a first order Taylor series, for small variations. The MIT code is freely distributed, however a few hardware commercial implementations of the MM are available, paving the way to the real-time analysis. If the video is long-lasting, the required computer time may be a major problem. Other physical limitations, such as the ones regarding illumination, shadows, camera unwanted vibrations, poor pixel resolution, low frame rate, presence of large motion, distance from the object, decrease severely the quality of the motion magnification and should be taken into account in order to achieve good quality results. In particular, the scene illumination should remain constant, as changing the background light could produce apparent motions.

Fig. 1. The masonry structure on the shaking table at the beginning of the test.

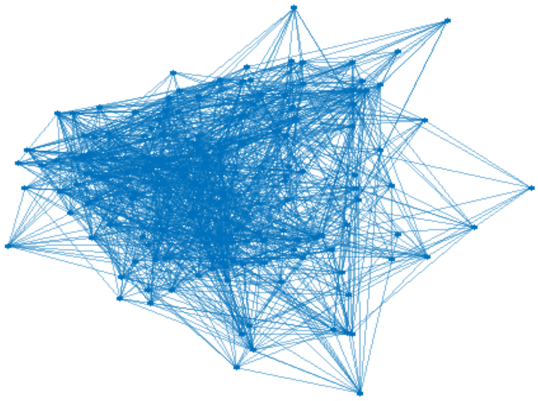

Fig. 2. A graph obtained from magnified video.

III. EXPERIMENTS

A scale model of the cross vault of the mosque of Dey in Algiers [15], was tested at the ENEA Casaccia Research Center shaking table facility (Figure 1). The model underwent several shaking table tests and finally was brought to collapse [3]. In the video of the final destructive test, the structural collapse occurs at time of about 6 s. The video was acquired at a frame rate of 30.3 frames per second (fps) and at pixel resolution of 640 x 480. The overall number of frames available before collapse was 182. During these few seconds, a number of control parameters are evaluated to find significant variations suitable to be used as an alert.

There are two main problems: first, how to transform a frame into a graph, second, the structural deformation could be too small, producing too small variations in the parameters to be significant. The second point is solved by the MM that amplifies the tiny displacements in the video, without major topological modifications. The first issue is addressed realizing the graphs according to a simple "distance" rule:

$$d_{ij} < threshold \qquad k = 1 \ldots 182. \qquad (3)$$

where i and j are pixel of the k-th frame and d_{ij} is the difference of the intensity values of i and j [8, 12, 13]. If (3) is true, then an edge exists between node i and node j of the graph, see Figure 2. Albeit simple, the rule (3) preserves all the necessary topological conditions.

Clearly, not all the pixels are relative to the structure, for example those on the image background. They could interfere with the construction of the graph, but since are "still", their action on the graph may be discarded, if the lighting is properly arranged. Moreover, only the connected component of the graph is considered. Despite this, the rule (1) produces a certain amount of calculations, relevant for real-time applications.

Note that the transformation problem frame-to-graph is transferred to the determination of a correct *threshold*. Unfortunately, its exact determination is still an open issue. Here we resort to the maximization of a sort of cost function of the graph parameters. Now we may calculate the graph parameters. As said before, we have examined the degree, the closeness and the eigenvalues spectra (but other choices are possible). The graph parameters are calculated for every of the 6.0 *s* * 30.3 fps, that is for each $\mathbf{L}_k$, and $\mathbf{A}_k$, giving rise to a time-series for any given parameter. We look for a significant [12, 13] variation in these time series, enough before the breakdown visibly takes place. The best results have been obtained by the inverse algebraic connectivity.

IV. RESULTS

In this section, we examine the data obtained from the experiment on the shaking table. As said before, for each graph G_k, that is for every matrix $\mathbf{L}_k$ and $\mathbf{A}_k$ we calculate a value for degree, closeness and eigenvalues spectra. Thus we have some time-series for $k = 1 \ldots 182$.

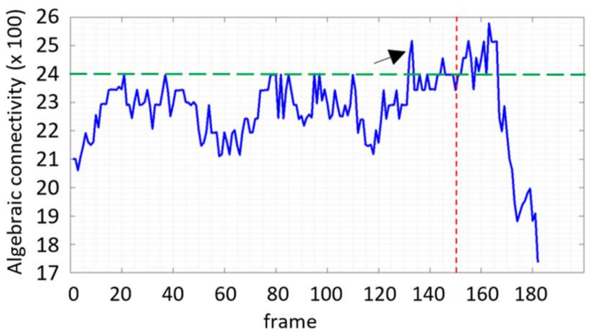

Fig. 3. Inverse of the algebraic connectivity. The arrow indicates a peak over a threshold (green dotted line). The dotted red line indicates the beginning of the collapse (152th frame).

Fig. 4. Complete activation of collapse mechanism (time t = 6.0 s).

Figure 3 shows that the inverse of the algebraic connectivity remains below a threshold of 0.024 until time t = 4.39 *s*, corresponding to the 133^{th} frame in the magnified video, when it has a peak of 0.025. The above threshold can be used to alert that the network is quickly losing connectivity, meaning the structure is losing rigidity. The warning signal can be issued just a few tenths of a second before initiation of collapse, which occurs at t = 5.02 s. After time *t* = 5.40 *s* (163^{th} frame), the deformations in the structure are so large that the topological properties do not hold anymore, as well as the algebraic connectivity. At *t* = 6.00 *s* (182^{th} *f*rame) the collapse is definitely activated, as shown in Figure 4.
The threshold provides an early alert of only 0.6 *s* before the collapse. However, using a camera with higher frame-rate/pixel resolution this time interval could be increased. Playing the magnified video in the slow-motion mode, these situations appear clearly. The other parameters show similar performances, but with several false alarms, therefore are not reported here. However, during the review phase of the paper, it was suggested to improve the prediction combining several parameters. Actually, this is possible, and, if the spectral calculation are too much cumbersome, this strategy could even become mandatory.

V. CONCLUSIONS

In the present work the possibility to outline a new method based on graphs to define a collapse alert system by studying the magnified video of a shaking table test of a masonry structure was explored. The experimentation was carried out through seismic tests intended to reproduce the effects of earthquakes on an ancient masonry structure, until its final collapse. The innovative approach exploited the potential of graph topological invariants extracted from the video motion magnification of the structure under earthquake shaking. Each frame was transformed in a graph, then some standard graph parameters were calculated. The inverse of the algebraic connectivity was identified as a possible indicator to predict the collapse, intended as an abrupt change of the structure shape. In the experiments, it predicted the collapse 0.6 s in advance. On the basis of such encouraging results, the Fiedler eigenvalue should be considered for further studies with more data and experimental cases, to point out a proper alert signal for structural collapses. Although the anticipation time provided by the proposed alert method may be not sufficient to save lives, it could be used to activate protection systems.

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