2023 IMEKO TC-4 International Conference on
Metrology for Archaeology and Cultural Heritage
Rome, Italy, October 19-21, 2023

Design Analysis: Research experiences from Alexandrian manuals to Imperial Architecture

Silvia Bertacchi[1], Francisco Juan-Vidal[2], Filippo Fantini[1]

[1] Alma Mater Studiorum University of Bologna, Dipartimento di Architettura, viale del Risorgimento 2, 40136 Bologna (Italy), silvia.bertacchi@unibo.it; filippo.fantini2@unibo.it
[2] Universitat Politècnica de València, Instituto Universitario de Restauración del Patrimonio, Camino de Vera, s/n, 46022 Valencia (Spain), fjuan@ega.upv.es

***Abstract* – Over the last 10 years, an international multidisciplinary research group formed by the Department of Architecture of the University of Bologna, the Instituto Universitario de Restauración del Patrimonio (IRP) of the UPV and the direction of Villa Adriana and Villa d'Este Institute, has carried out a series of surveys and analyses on well-known buildings at the Hadrian's Villa. The paper summarises the results of these studies, that besides documentation, are focused on the interpretation of ancient texts related to the field of architectural design. This research has led to the development of a method for the study and dissemination of ancient design combining a technological component (acquisition with laser scanning and photogrammetric devices), a philological one (integration of theoretical and technical ancient literature), and finally the elaboration of 2D/3D synthesis models capable of expressing and summarising the design steps in a scientific output (from point clouds to 3D digital polygonal models).**

Keywords – Hadrian's Villa, Heron of Alexandria, Vitruvius, Archimedes, design analysis, 3D digital models.

I. INTRODUCTION

Scientific considerations on the use of digital models for the study of archaeology since long ago has led to the definition of 3D models for scientific purposes [1] that, as a "laboratory", allow scholars to test and analyse the rules of architectural design methods by means of including the results of both surveys and construction knowledge. In the paper a series of case studies are then provided to illustrate development and fundamentals of an analysis method that, although still being improved, has led to relevant results that hopefully will strengthen the relationship between architects, historians, and archaeologists.

The present research takes the steps from a first case study, a rare and well-preserved artifact, the so-called "model of stadium" [2], found in the Great Baths at Hadrian's Villa. This is an architectural *maquette* of a building designed for the Hadrianic mansion, but which most likely was never built.

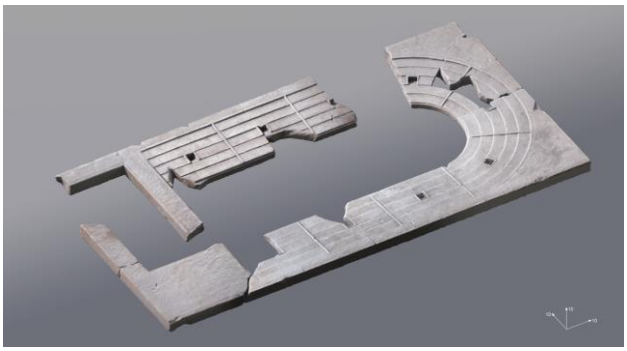

Fig. 1. Axonometry of the digital model of the maquette carved in a marble slab representing the model of stadium. It is to be noted how the chisel work is surrounded by a smooth edge running along all the main parts of the layout. Illustration by F. Fantini.

Even though "left on paper", this artifact is the only one that bears witness to the design phase within the cultured and refined patrons in the Hadrianic reign.

The scale model carved on a marble slab clearly explains a series of problems that the ancient architect would have had to face and express, unambiguously, before the Emperor, who, as the sources tell us, was competent in architectural design, so much so that he himself developed innovative and complex vaulted spaces [3].

The original use of a hybrid form of representation, halfway between planimetric drawing and three-dimensional architectural model (Fig. 1), places the ways of expression of this object very close to another model (also partial and apparently incomplete) found in Ostia Antica [4, 5, 6] (Figs. 2 and 3).

The priority, in both cases, is to express distribution, functions, dimensional relations and design requirements.

But there is more, especially in Hadrian's model, namely the need to explicitly highlight a module, i.e., a quantity that is repeated throughout the plan: a "background grid" that derives from a skilful fitting of geometric constructions based on circles and figures determined with the formulas included in the manuals – dating back to the first century A.D. – for calculating areas and volumes.

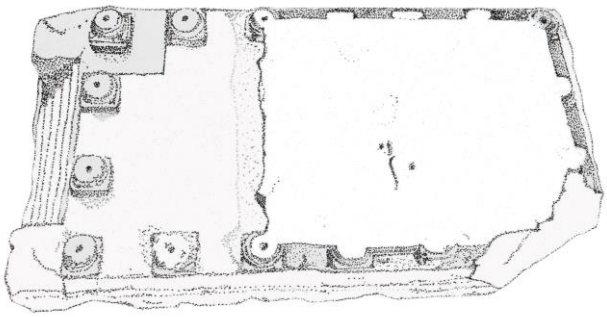

Fig. 2. Model of a temple from Ostia Antica. Illustration by S. Bertacchi after [6, p. 125].

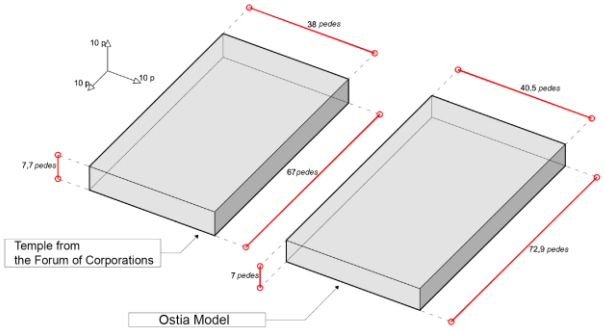

Fig. 3. Dimensional comparison between the marble model (left) and the Temple in the Forum of Corporations in Ostia Antica (right) which is of a similar type (assuming the model scale 1:32). Illustration by S. Bertacchi.

The way in which the ancient architect decided to underline the presence of this module, the *koilòn métron* as Heron defines it in the "Definitiones" [7], provides an important technical advice, evident only to the experts.

II. A WORKING METHOD FROM THE MODELS

Both models therefore have not just an exhibition value, commonly found in such objects, but goes much further. In particular, the stadium *maquette* does not merely display the modular structure that is intended to give metric coherence to the entire building, but it also emphasises further aspects that were supposed to be of major importance in the communication of the design, such as the perimeter of the empty flat space needed for the new construction.

Probably this is why the outer edge of the marble slab is emphasised, as well as the perimeter of the elevated parts by means of a thin, smooth band that interrupts the chisel work. And yet, the covered and uncovered areas (tiers of seats, perhaps an adjoining volume that can be interpreted as a portico, the arena for games and a possible courtyard/peristyle) are respectively represented in the form of solids and voids, i.e., marble areas alternating with actual holes in the marble slab.

Hidden from the perceived form, but nonetheless present, there are also relationships that are worth dwelling on as they are part of a deeper understanding of the term *symmetria* used by both Vitruvius and Archimedes, that can often be misleading [8].

The concept of *symmetria* is often simplified by associating it with the use of modular design. Actually this term indicates a quality and not a tool made available to the architect.

The Syracusan mathematician in his first book "On the Sphere and the Cylinder" uses the same term in reference to the discovery of simple proportions linking areas and volumes of the sphere, cylinder, and cone. In other words, if two geometric figures have areas (or volumes) linked by simple proportionality, they can be said to be "symmetrical". Whether this property is achieved through the use of modules is another matter that concerns the technique Vitruvius defines as *ordinatio* [9].

But if *symmetria* is a problem of areas or, rather, of ratio between areas, to which quantities must we refer for a planimetric reading of the ancient project?

It is useful to consider the ancient building as the answer to a series of needs that can be expressed by means of standard areas, used in the Roman context to manage the territory, to hierarchically subdivide and distribute it.

The best insight to this issue found in the written sources is provided by Columella [10] who – to our knowledge – lists the widest range of standard surfaces compared to other authors [11].

In this sense, an interesting sub-module of the *actus quadratus* (120x120 *pedes constrati*) called *clima* (60x60 *pedes constrati*) turns out to be very "close" to the areas that could be used to define the plan of the ancient building (17.7 metres on each side, just over 300 m^2).

So, on the basis of a series of hypotheses regarding the scale of the stadium model [12], a coherent system of interpretation of the areas used for the three sectors of the tiers of seats was determined. In this hypothesis each section (one curved and two straight) would be equal to one *clima* (3600 *pedes constrati*) (Fig. 4).

Subsequent research on second-century A.D. architecture then focused on the theme of centrally planned spaces, particularly characterized by the presence of domes in *opus camenticium* [13, 14].

From this point of view, Hadrian's architectural production is particularly rich in spectacular examples that show the plastic and scenic qualities of the pozzolana-combined vaults [15].

The complex design of buildings with such complex and expensive domes (in terms of materials and construction logistics) required painstaking planning that relied on precise computations. In particular, the control of curved surfaces both two- and three-dimensionally (from the *cavea* of a theatre to the construction volume of a dome). What is certain, however, is that such shapes constituted for the ancient architect a source of quite considerable numerical problems, linked to the use of π [16].

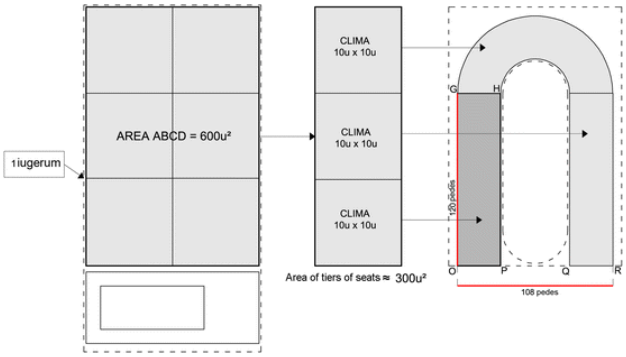

Fig. 4. Planimetric analysis of the so-called stadium model from Villa Hadrian's Villa. Illustration by F. Fantini.

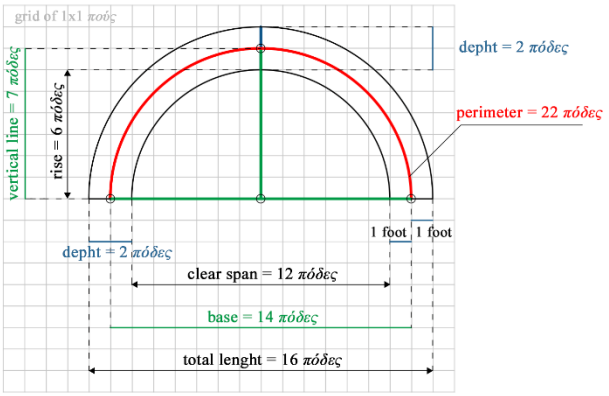

Fig. 5. The interpretation of Heron's formula for the calculation of the constructive volume of an arch (Stereometrica II, V, 29). Illustration by S. Bertacchi.

In this respect, the clarifying reading of Heron's writings in Heiberg's critical edition [7, 17] provides extremely important keys to understanding how the architect could calculate the *cavea* (a semi-circular annular ring) for the purposes of determining the number of spectators to be seated [11]. The problem is also posed in very similar terms for calculating the construction volume of arches (Fig. 5) and cupolas. Scholars of the subject [16, 18, 19] agree on two basic aspects that we find repeatedly present in Hero's texts, namely on the extensive use of the so-called side and diagonal number progressions (of the square inscribed or circumscribed in centric plans) and on the use of diameters equal to 7 feet (or modules) and multiples of 7.

III. APPLICATION ON CASE STUDIES

On the topic of standard areas used for a pre-dimensioning of the seats in buildings for entertainment, an example from the 1st century B.C. is particularly significant as it shows an evident adherence to the model of the stadium at Hadrian's Villa. The layout of the amphitheatre of Caesarea of Mauritania, built in the 1st century B.C. by Juba II [12] suggests that, in the initial phases of the design of buildings for spectacles, pre-dimensioning criteria were centred on the use of numerically "convenient" areas, such as the *clima*: as a matter of fact the number 60 admits a large number of dividers, easily adaptable with the determination of a flexible module, that could be certainly associated with the number of spectators (Fig. 6).

Even residential buildings, called "non-aulic", such as the *Antiquarium* located on the west side of the Canopus pool of Hadrian's Villa (Fig. 7), seem to confirm this logic of standard areas, then subdivided according to modular grids, completely in line with the design method of buildings for entertainment, although with the simplified condition of not presenting curvilinear elements [20].

Given the greater planimetric simplicity of the complex (Fig. 8), in the latter case emerges more clearly the application of design methods based on the concept of the overall subdivision of the built areas, *symmetria* with uncovered spaces, and the use of standard areas (*actus*, *clima*, *scripolum*).

A third example is related to the design of complex domes based on the geometry of the sphere intersecting spheroidal surfaces (sail vaults) which – in the writings attributed to Heron – is referred to as "trikentron" [14]. The Golden Square Vestibule is a building that would serve as an archetype for many other later examples of octagonal halls covered with "compound domes": a semi-sphere intersects with an octagonal prism, the resulting semi-circular lunettes on the flat sides (Fig. 9) are connected to the sphere by *trikentrons*. The formulas found in Stereometrica II provide all the tools needed to calculate surfaces and volumes of such an articulated spatial solution.

In particular, the "rule of 7" [21] is easily found in the hemispheric dome since the intermediate semicircular profile of the structure made of *opus caementicium*, tuff and brick has a diameter of 35 *pedes*: a number easily divisible by 7 and thus able to simplify the calculation of the dome's volume. One question, still open in scholarly debate, is the one of how the ancient architect would reconcile this set of formulas with the graphic tools habitually employed for the planimetric layout of the building. The construction known as "*ad quadratum*" (Fig. 9) is evidently used extensively in circular and octagonal buildings to create a pattern to be followed in the planimetric layout (*ichnographia*), which, in turn, will have to be harmonised with the absolute must of using modules (*symmetria*).

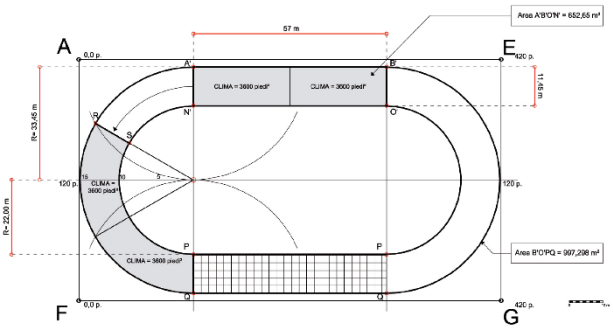

Fig. 6. Planimetric analysis of the amphitheatre of Caesarea of Mauritania. Illustration by F. Fantini.

Fig. 7. 3D model from the laser scanner survey of the Antiquarium on the west side of the Canopus pool at Hadrian's Villa and fitting areas (3 actus). Illustration by S. Bertacchi.

IV. CONCLUSIONS

Vitruvius repeatedly expresses in his treatise the need to refer to rational quantities and possibly whole numbers of modules obtained through calculations and geometric constructions performed with the compass, i.e., forcing "*ad abundantiam*", the result of the calculations performed through similar handbooks to those ascribed to Heron. The method proposed here is based on this premise and can be divided into five priority aspects to be taken into account when performing the design analysis of ancient buildings:

1. research of sequential elements to derive the module present in the whole building;
2. exam of the existing relationship between the module and the standard area measurements used in the imperial age (*jugerum, actus, clima*);
3. check for simple numerical relationships that link the parts of the building layout (covered areas/uncovered areas);
4. for curved layouts, with special regards to centric plans, research of a pattern based on inscribed and circumscribed circles with simple geometric figures such as triangles and squares (*ad quadratum* scheme), to be adjusted to previously obtained modules. In general, the final building measurements are the product of an overhaul of pure mathematical calculations or geometric constructions;
5. verification of the presence of modules or measures that are multiples of 7 to understand which quantities (volumes, areas) the ancient architect had to calculate to make the project executive.

This kind of study has a twofold objective, on the one hand to find a basis for a fruitful dialogue with archaeologists through a philological investigation of technical texts.

On the other, to regulate, through a clear codification, a type of research on ancient design that is often the subject of empirical graphic analyses, lacking a real causal link between forms and functions/requirements.

Hadrian's Villa is an archaeological site known for the great formal variety of buildings dedicated to conviviality and well-being (*triclinia*, baths, buildings for spectacles), but also for the presence of more functional constructions with a simple layout such as the residential ones and those for administration [22]. The coexistence of complex monumental buildings, especially the dome-shaped ones with a mixtilinear plan, with the more elementary non-aulic ones, allows us to see closely and clearly the application of a univocal and rigorous design method, which in the more eclectic buildings becomes complicated, leading to solutions that are not always optimal. It is precisely this co-presence that allows scholars to recognise in a single ensemble the rule and its exception with great clarity, provided that, however, up-to-date surveys and multidisciplinary collaborations are available.

The extension of surveys carried out with advanced tools also to other buildings belonging to the same cultural landscape, such as those in Baiae (Temple of Venus in Campania, near Naples) and in Rome (Horti Sallustiani) would allow for an even deeper understanding of the design methods most widely used during the Hadrianic reign which, despite its high originality, did not abandon the core of the classical design methods enunciated by Vitruvius. In an era in which research starting from digital surveys, then turned into 3D digital models, seems to be mainly directed towards the "transparent" representation of the data collected in order to obtain valid reconstructions [23], we believe it is fundamental to emphasise the relevance of knowledge integration to improve the process of understanding the design methods adopted in antiquity: an indispensable basis for any philologically reliable reconstruction [24].

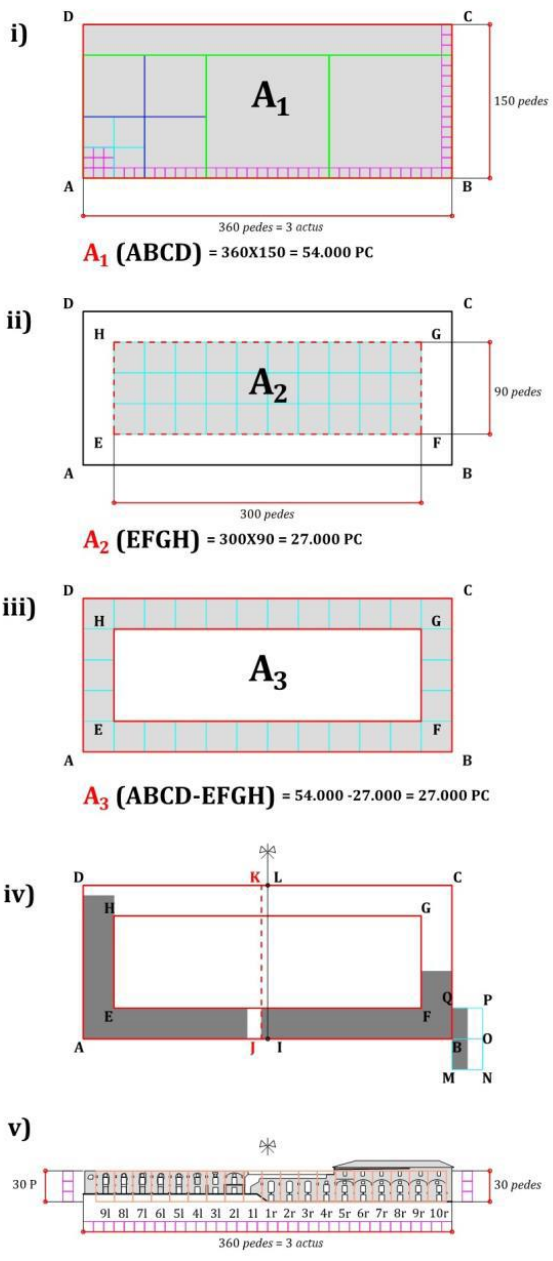

Fig. 8. Planimetric analysis of the residential building located in the west area of the Serapeum-Canopus Complex at Hadrian's Villa. Preliminary hypothesis for the plan of the original building (Roman units of measurement for surfaces: pedes, actus, PC=pedes constrati). i) Estimate of the global area of the upper building considering the plausible design of the plan (A1); ii) estimate of the courtyard according to preserved remains and excavations (A2); iii) possible covered area of the building (A3); iv) current existing upper structures (dark grey hatching); v) proportions, height difference and modules of the eastern front. Illustration by S. Bertacchi.

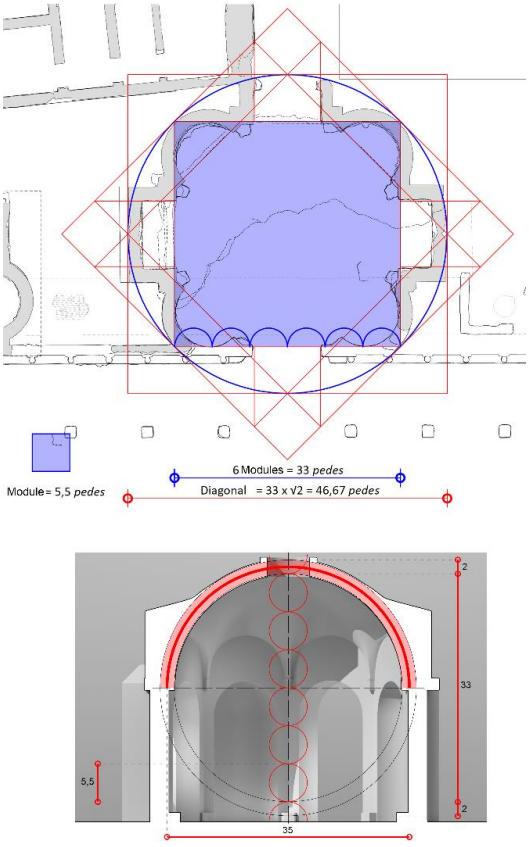

Fig. 9. Design analysis of the access Vestibule at the Golden Court: ichnographia (ad quadratum) and orthographia (Heron's formulas). Illustration by F. Juan-Vidal.

ACKNOWLEDGEMENTS

Authors would like to thank Andrea Bruciati, Director of the *Villae*, Istituto Autonomo Villa Adriana e Villa d'Este, and Benedetta Adembri for the valuable support and scientific collaboration.

All authors contributed to the writing, review and editing of the paper. Filippo Fantini is author of Section I-*Introduction*. Silvia Bertacchi is author of Section II-*A working method from the models* and III-*Application on case studies*. Francisco Juan Vidal is author of Section IV-*Conclusions*.

All authors read and approved the final manuscript. All images, 3D models and digital elaborations, unless otherwise specified, are by authors.

REFERENCES

[1] A.Ippolito, F. Borgogni, "I modelli 3D nei rilievi di architettura", in E.Chiavoni, M.Filippa (eds.), Metodologie integrate per il rilievo, il disegno, la modellazione dell'architettura e della città, Gangemi, Roma, 2011, pp. 71–77.

[2] B.Adembri, L.Cipriani, F.Fantini, "Illustrare,

pianificare e costruire nel mondo antico: casi di studio da Villa Adriana", in C.Inglese, A.Pizzo (eds.), I tracciati di cantiere Disegni esecutivi per la trasmissione e diffusione delle conoscenze tecniche, Gangemi, Roma, 2016, pp. 90–107.
[3] A.Viscogliosi, "L'architettura Adrianea: di Adriano, per Adriano, sotto Adriano, dopo Adriano", in R.Hidalgo Prieto, G.E.Cinque, A.Viscogliosi, A. Pizzo (eds.), Adventus Hadriani, Investigaciones sobre arquitectura adrianea, L'Erma di Bretschneider, Roma, 2020, pp. 11–36.
[4] E.Salza Prina Ricotti, "Villa Adriana. Il sogno di un imperatore", L'Erma di Bretschneider, Roma, 2000.
[5] P.Pensabene, "Modello templare ostiense in marmo lunense dal Collegio degli Augustali", Rivista dell'Istituto Nazionale d'Archeologia e Storia dell'Arte, S.III, XIX-XX, 1996-1997, Roma, 1998, pp. 148–151.
[6] C.Caprino, "Plastico marmoreo di uno stadio nella Villa Adriana a Tivoli", Rivista dell'Istituto Nazionale d'Archeologia e Storia dell'Arte, S.III, XIX-XX, 1996-1997, pp. 113–151.
[7] J.L.Heiberg, "Heronis Alexandrini opera quae supersunt omnia", vol.IV: Heronis definitiones cum variis collectionibus. Heronis quae feruntur geometrica, Teubner, Stuttgart, 1914 (rpt. 1976).
[8] A.Frajese, "Opere di Archimede", UTET, Torino, 1974.
[9] P.Gros (ed.), "Vitruvio. De architectura", Einaudi, Torino, 1997.
[10] C.Carena (ed.), "Lucio Giunio Moderato Columella, L'arte dell'agricoltura e libro sugli alberi", Einaudi, Torino, 1977.
[11] C.Bianchini, F. Fantini, "Dimensioning of Ancient Buildings for Spectacles Through Stereometrica and De mensuris by Heron of Alexandria", Nexus Network Journal, vol.17, 2015, pp. 23–54.
[12] Y.Chennaoui, F.Juan Vidal, F.Fantini, "Architectural models and urban planning", Proc. of the 17th International Conference on Cultural Heritage and New Technologies 2012, Museen der Stadt Wien-Stadtarchäologie, Vienna, 2013.
[13] L.Cipriani, F.Fantini, S.Bertacchi, "The Geometric Enigma of Small Baths at Hadrian's Villa: Mixtilinear Plan Design and Complex Roofing Conception", Nexus Network Journal, vol.19, 2017, pp. 427–453.
[14] A.Roca, F.Juan-Vidal, L.Cipriani, F.Fantini, "Heron's Legacy: An Example of Ancient Calculations Applied to Roman Imperial Architecture", Nexus Network Journal, vol.25(1), 2023, pp. 185–192.
[15] C.F.Giuliani, "L'edilizia nell'antichità", Carocci Editore, Roma, 2018.
[16] W.Fuchs, "Studio geometrico della forma architettonica del tempio di Vesta ricostruito nel Foro Romano", AEDES VESTAE. Archeologia, Architettura e Restauro, Atti della Giornata di Studio, Gangemi Editore, Roma, 2022, pp. 77–86.
[17] J.L.Heiberg, "Heronis Alexandrini opera quae supersunt omnia", vol.V: Heronis quae feruntur stereometrica et de mensuris, Teubner, Stuttgart, 1914 (rpt. 1976).
[18] H.Svenshon, "Heron of Alexandria and the Dome of Hagia Sophia in Istanbul", Proc. of the 3rd International Congress on Construction History, NEUNPLUS1, Berlin, 2009, pp. 1887–1394.
[19] C.Conti, G.Martines, "Hero of Alexandria, Severus and Celer: Treatises and Vaulting at the Nero's Time", in A.Sinopoli (ed.), Mechanics and Architecture between epísteme and téchne, Edizioni di Storia e Letteratura, Roma, 2010, pp. 79–96.
[20] A.Ottati, S.Bertacchi, B.Adembri, "Integrated methods for documentation and analysis of archaeological heritage: the residential building along the western side of the Canopus at Hadrian's Villa. initial results and research perspectives", SCIRES, vol.8(2), 2018, pp. 85–106.
[21] W.Fuchs, "The New Theory of the Metrological Framework of the Pantheon", Nexus Network Journal, 25(1), 2023, pp. 103–110.
[22] E.Eramo, "Gli edifici residenziali "non aulici" nella Villa Adriana di Tivoli: caserma dei Vigili, Hospitalia, Triclinio Imperiale, novità e indirizzi di analisi", in R.Hidalgo-Prieto, G.E.Cinque, A.Viscogliosi, A.Pizzo (eds.), Adventus Hadriani, Investigaciones sobre arquitectura adrianea. L'Erma di Bretschneider, Roma, 2020, pp. 607–632.
[23] F.I.Apollonio, F.Fallavollita, R.Foschi, "The Critical Digital Model for the Study of Unbuilt Architecture", In F.Niebling, S.Münster, H.Messemer (eds.), Research and Education in Urban History in the Age of Digital Libraries. UHDL 2019. Communications in Computer and Information Science, Springer, Cham, vol.1501, 2021, pp. 3–24.
[24] I.Bassoli, F.Fallavollita, W.Fuchs, "Hypothesis of reconstruction of the Roman theater of Urbs Salvia", SCIRES, vol.12(1), 2022, pp. 151–164.