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Analytical Models and Magnetic Position Systems

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***Abstract* – In this work, we promote the use of analytical solutions for magnetic position system design and analysis, which has become extremely convenient through the development of the open-source Magpylib computational package. We discard three common arguments against this ansatz by showing that analytical models are suitable for dealing with complex shapes, inhomogeneous magnetizations and even material interactions. Accuracy of analytical models is discussed, and the computational performance is demonstrated with three examples, a complex shape, an inhomogeneous magnet, and the calibration of a position system experiment. We find that analytical models can be powerful tools in this context.**

I. INTRODUCTION & MOTIVATION

Magnetic position systems are commonly used to track the motion in mechanical machinery by measuring the relative position between a permanent magnet and a magnetic field sensor. Such systems are widely used in industrial applications due to their robustness, versatility and potential for miniaturization [1, 2]. However, while some basic design patterns for common applications like rotary encoders, linear position systems, and joystick motion are established, advances in sensing and magnet technologies, a large and competitive market, as well as the development of novel applications require a constant modelling effort for improving and optimizing designs.

To meet this demand, commercial finite element environments like Ansys or Comsol are the tool of choice in engineering circles. While finite element computations are well-developed powerful tools, their main disadvantage is the computation speed, which makes it difficult to sweep through design variations and impossible to find optimal designs, when more than two or three variables are involved. Typical position systems deal with 30 - 100 variables that include parameters of geometry, magnetic material, sensor and tolerances.

In this paper, we endorse analytical models for designing a large variety of position systems due to their computational performance and the recent development of the Magpylib package [3], which gives effortless access to performant implementations. We discuss advantages, disadvantages, and accuracy of such models and discard several common arguments against their use that include "only simple geometries", "only simple magnetizations", and "no material interaction".

II. ANALYTICAL MODELS

A permanent magnet corresponds to a spatial magnetization distribution $\mathbf{M}(\mathbf{r})$. The field of such a distribution is simply the sum of the fields of the individual magnetic moments, which can be expressed in a macroscopic continuum approximation [4] as

$$\boldsymbol{B}(\mathbf{r}) = \frac{1}{4\pi}\int \frac{\boldsymbol{\nabla}'\cdot \boldsymbol{M}(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}V. \quad (1)$$

These integrals can be solved analytically in many cases depending on geometry and magnetization. Magnetic field expressions are then reduced to closed forms (polynomial, rational, trigonometric,...), and expressions such as elliptic integrals that are easily evaluated. Solutions that can be found in the literature include cuboid [5], cylinder [6, 7], cone [8], triangle [9], and other basic shapes with homogeneous magnetizations, of which most are implemented in Magpylib. Finding new solutions is an on-going effort.

One major argument against analytical solutions is that only simple geometric forms are possible. This statement ignores the possibilities offered by discretization and superposition. In magnetostatics, arbitrary complex forms can be constructed by combining simple base geometries. While this hinders the performance, in most cases it is possible to find a good compromise in terms of discretization finesse, required accuracy, and computational speed. In this context, the triangle solution must be mentioned, because it does not require a volume mesh, but only a surface approximation. An example of a complex magnet form realized with a triangular mesh is shown in Fig. 1. A B-field computation with Magpylib of this body including 1000 triangular facets takes 1.9 ms on an Intel Core i5-1235U without multiprocessing.

It should also be noted, that for many applications the simple base geometries are sufficient. This is demonstrated by the industry standard DIN SPEC 91411 [10], where these forms are described in detail. The reason for this are mostly off-the-shelf solutions offered by magnet man-

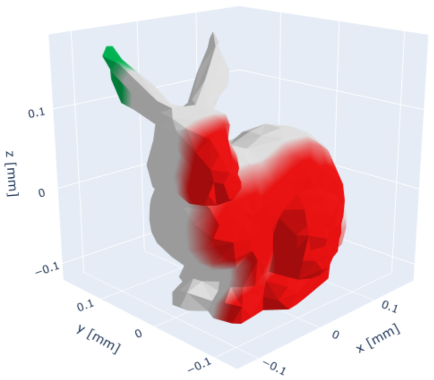

Fig. 1. Complex magnet form realized with triangular surface mesh.

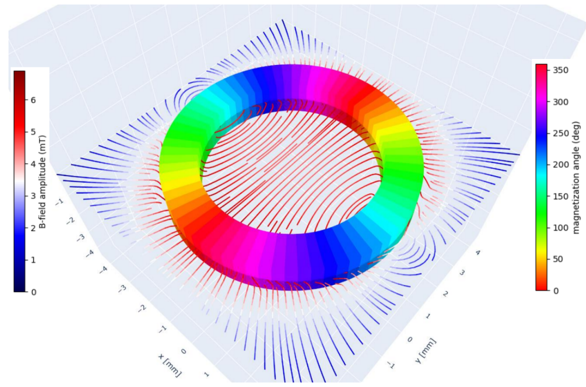

Fig. 2. Approximation of Halbach magnetization and resulting magnetic field.

ufacturers, low costs in fabrication and development, and superior magnetic properties when compared to special geometries realized, e.g., by milling and injection molds of polymer bound magnets [11].

The second argument against using analytical solutions is that magnetizations are often inhomogeneous, for which very limited solutions exist. Specific examples are magnetic multipole rings with a low number of poles, or magnetic scales with thick magnetic material layers that are not magnetized all the way through. While numerical solutions based on direct integration are possible [12], discretization methods based on analytical solutions should not be discarded. Any inhomogeneous magnetization distribution can be approximated by splitting the magnet into small pieces, each with a different homogenous magnetization. An example of this is shown in Fig. 2, where a continuous Halbach cylinder is approximated by a discretized version of 61 base cylinder tile geometries [7]. The homogeneous field on the inside and the fast decay of the field on the outside of the cylinder are clearly visible. This computation of the B-field at 400 grid positions with magnet comprised of 61 cells took less than 5 s on an Intel Core i5-1235U mobile CPU without multiprocessing.

In summary, it can be said that discretzation methods enable analytical solutions for complex forms and inhomogeneous magnetization. The downside is that computation performance is reduced because the field of many cells must be computed. However, this is often mitigated by the ultra-fast computation times of individual base geometries that are about 1-100 µs and the remarkable possibilities for multiprocessing that are discussed below. In addition, the discretization approach is in line with modern open-source meshing tools [13], that generate efficient meshes of complex bodies without much effort.

III. MATERIAL RESPONSE AND ACCURACY

It is often stated that analytical methods cannot be used to compute the effects of material interaction. What is actually meant is that the inhomogeneous magnetization distributions resulting thereof are typically not known. When using finite element tools, the user input is not the magnetization distribution, but a known initial state and material parameters. For permanent magnets, the initial state is “the instant of their magnetization”, and for ideal soft-magnetic materials it is simply zero. The FE environment then solves the magnetization problem, and computes the B-field at the same time.

There are two major effects that must be considered: Demagnetization in hard-magnetic materials ($\mu_r \ll 5$), and magnetization of soft-magnetic materials ($\mu_r \gtrsim 500$). The first effect is relatively weak because of the low permeabilities of most modern hard magnetic materials. For example, sintered NdFeB typically has a value of $\mu_r \leq 1.05$. The relative error resulting from demagnetization with such materials are estimated in [14] for cubical magnets as 10^{-2}, and less than 10^{-3} when the homogeneous part is compensated. In comparison, FE errors as low as 10^{-4} can only be achieved by immense computational effort. The effect depends on the magnet geometry and on the distance from the magnet. In addition, softmagnetic materials are generally avoided in magnetic position systems because they give external stray fields a position dependence. In combination, this already opens a wide range of applications for working with analytical models in which demagnetization effects play a negligible role.

However, to compute the magnetization distribution of hard and soft materials with arbitrary initial states, it is possible to rely on the Magnetostatic Method of Moments [15], which is a discretization method that solves the interactions between individual cells analytically [16]. The method is demonstrated here using a point-matching interaction (Magpylib H-field computation) to compute the demagnetization of a cuboid magnet with an unfavorable

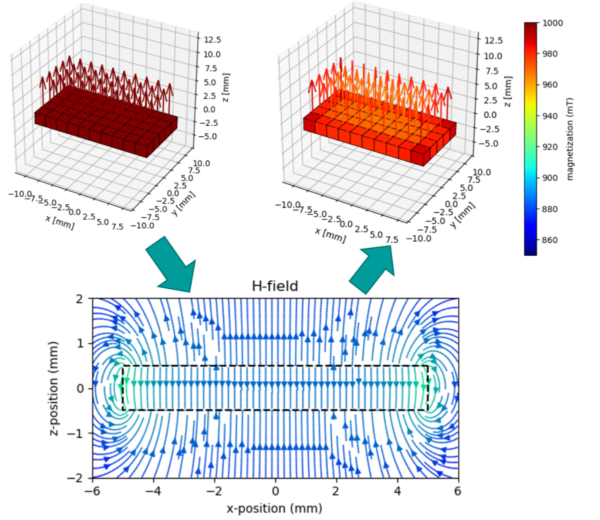

Fig. 3. material response.

shape of 5 x 10 x 1 mm^3 magnetized in z-direction with a remanence field of 1000 mT and an isotropic relative permeability of $\mu_r = 1.05$. The result is shown in Fig. 3. The initial state is a homogeneous magnetization, which creates a strong opposing field inside the magnet which resulting in a change of the magnetization state. One can observe an overall reduction of the magnetization by ≈ 2 %, superposed with a small variation between sides and center of the magnet slab. This method can also be applied to treat softmagnetic bodies.

IV. ADVANTAGE AND PROOF

Magnetic field computations based on analytical solutions have unparalleled computational speed. Individual data points can be obtained in microseconds, which is typically 6-9 orders of magnitude faster than corresponding FE computations, and enables the use of standard global optimization strategies in 10-100 dimensional spaces. There are two types of optimization problems that are regularly encountered with magnetic position systems: (i) finding optimal system designs, which requires the minimization of complex cost functions including system construction constraints, with an excellent example shown in [14], and (ii) the fitting of experimental data to understand experimental tolerances, which is demonstrated below.

Specifically, evolutionary black box optimizers were shown to be highly synergetic with analytical methods [17]. This synergy can be exploited maximally when parallellization is an option. Here, an evolutionary optimizer can make use of many unrelated input points in each generation, which can all be computed in parallel on multiple cores or even computational clusters. The license-free Magpylib is especially useful when computational upscaling is attempted.

We demonstrate the calibration of an experiment in Fig. 4. In this figure, we see a classical linear position

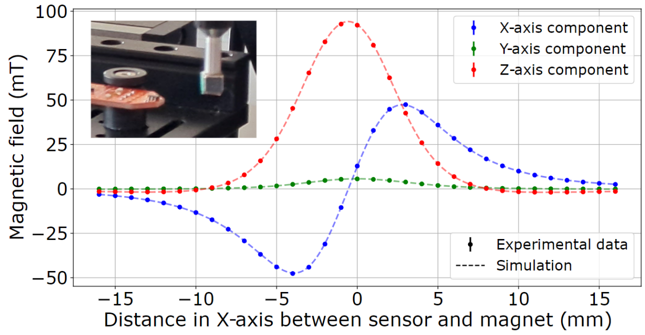

Fig. 4. material response.

system experiment, where a magnet moves linearly with respect to a sensor. The most difficult part in such an experiment is aligning magnet and sensor properly. We fit an analytical model to the experimental data obtained with an Infineon TLE493D 3D Hall sensor, leaving magnet magnetization, sensor displacement, orientation, offset and sensitivities as fitting variables. With 33 data points and 18 fitting variables, a least mean square fit took less than four minutes on an Intel Core i5-1235U mobile CPUs using the SciPy differential evolution algorithm [18, 19] with standard settings and eight workers. The figure shows the excellent alignment between experiment and theory with a relative error below 2.85 %.

V. CONCLUSION

In this paper, we have discarded three classical arguments against the use of analytical solutions for magnetic field computation by demonstrating that analytical models can deal with arbitrary shapes, inhomogeneous magnetizations, and even material response computation. We have demonstrated the performance of analytical solutions by computing the magnetic field of a complex shape and a Halbach cylinder, and by solving a highly complex fitting problem with 18 degrees of freedom. While analytical models are not the answer for every problem, they should be considered as powerful design and fitting tools, at least in pre-development stages to give good starting values to sophisticated yet slow FE-based optimizers.

VI. *

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