# Atomic density calibration at high-temperature

Erxat Arkin<sup>1</sup>, Jiqing Fu<sup>2</sup>, Qing He<sup>2</sup>, Jia Kong<sup>1</sup>

<sup>1</sup> Department of physics, Hangzhou Dianzi University, Hangzhou 310018, China <sup>2</sup> Magnetic quality laboratory, National Institute of Metrology, 100029 Beijing, China

*Abstract* – Calibration of atomic density in vapour cell is very important for high precision quantum metrology and fundamental research. Here we propose to use spin noise spectroscopy(SNS) method for density estimation which would be more reliable than the common density estimation methods based on absorption spectrum or thermodynamic equation. We show our test with different density estimation methods, and show the robustness of SNS method.

# I. INTRODUCTION

Atomic density of alkali vapor is one of the most important parameters of atomic ensemble, which has broad applications in quantum metrology, such as atomic magnetometer [1,2], atomic clock [3], atomic gyroscope [4], and also quantum noise investigation. Our knowledge of atomic density helps to increase measurement sensitivity and accuracy. It is even more crucial for quantum noise study topics that based on atomic interactions to generate quantum entanglement [5] where density calibration determines the standard quantum limit of the system. However, the common density calibration methods are not very suitable for high temperature atomic vapour.

A convenient way of knowing the atomic density is proposed by Alcock [6], based on thermodynamics,

$$n = \frac{1}{T} 10^{21.866 + A - \frac{B}{T}},$$
(1)

where A and B are parameters determined by atomic species. One can easily calculate atomic density with known temperature. However, it is hard to get accurate temperature of atomic vapour.. Firstly, normally we can only measure temperature at the surface of a vapor chamber which can not reflect the real temperature of the alkali vapor. Secondly, there are also limitations on the accuracy of temperature sensors. Absorption spectrum [7] is another way of measuring atomic density, which partly depends on temperature measurements. It gets density information by fitting the absorption spectrum of atoms, which makes it more reliable than thermodynamics methods. However at high temperature, too much absorption makes the spectrum saturated and therefore the fitting results are not reliable any more. Here we propose to estimate alkali vapor's density with spin-noise spectroscopy (SNS) [8], which does not rely on temperature measurement at all, and it can work for both low and high temperature conditions.

## II. ATOMIC ABSORPTION SPECTRUM

Absorption spectrum is relatively easy to operate. A laser beam passes through vapor cell with a frequency modulation closed to resonance. The intensity before and after the cell have to be recorded for further calculations. The absorption effects with a given length of alkali vapor are defined by

$$I(z) = I_0 exp(-\alpha(\nu, T)l), \qquad (2)$$

where  $\alpha$  is absorption coefficient that depends on frequency  $\nu$  of the incident light and temperature of the vapor chamber, which is defined as

$$\alpha(\nu, T) = \sigma(\nu, T)n(T). \tag{3}$$

where  $\sigma$  is absorption cross section and n is atomic density. We see that the density estimation relies on the measurement of temperature and beam power, but not solely decided by temperature measurement, therefore it can do a better job than thermodynamics equations.

At high temperature conditions, the atomic density is quite high, therefore the absorption effect is so strong that almost all probe power are absorbed on resonance, which makes the fitting results not very reliable. As can be seen in figure 1, above 90  $^{\circ}$ C, too much absorption covered the transition details.



Figure. 1. Absorption line versus frequency detuning of probing light under different temperature conditions.

## III. SPIN NOISE SPECTROSCOPY

SNS is a technique to study spontaneous fluctuation of spin system. There are several techniques that have been

applied to measure intrinsic fluctuation of the spin system, such as nuclear magnetic resonance [9,10] and magnetic force microscopy [11,12], but the most used probing technique is Faraday rotation measurement which can transfer information of spin noise to the polarization of an off-resonance probe light. The setup of SNS is relatively easy and very similar to absorption spectrum experiment, where we need to measure the polarization, instead of intensity, of probe beam.

Both photonic noise and atomic noise are included in the SNS signal. For atomic density calibration, we need to get atomic information. On time domain, it is hard to distinguish photonic or atomic noise, but on frequency domain, we could obtain atomic information since they have different frequency dependence. With fast Fourier transformation (FFT), we can get power spectral density (PSD) of the SNS signal, which can be described by Lorentzian function [13]:

$$S(\nu) = S_{ph} + S_{at} \frac{(\Delta \nu/2)^2}{(\nu - \nu_L)^2 + (\Delta \nu/2)^2}$$

where  $v_L$  and  $\Delta v$  are Lamour frequency and full-width halfmaximum (FWHM).  $S_{ph}$  is photon shot noise contribution to PSD, which is mainly determined by probe power P. The amplitude of Lorentzian line  $S_{at}$  is mainly contributed by atomic spin noise, which is defined as:

$$S_{at} = \frac{4G^2 \mathcal{R}^2 P^2}{\pi \Delta \nu/2} Var \Theta_{FR}$$

(5)

(4)

where *G* is transimpedance gain;  $\mathcal{R}$  is detector responsivity and  $Var \Theta_{FR}$  is the variance of Faraday rotation due to light atom interaction:

$$Var\Theta_{FR} = N \frac{\sigma_0^2}{A_{eff}^2} \kappa^2, \tag{6}$$

Here  $\kappa$  is a parameter determined by detuning frequency of probing light and atomic property.  $\sigma_0$  and  $A_{eff}$  are on resonance cross section and effective beam area. *N* is the number of atoms that interacts with probing light, which is the parameter we want to estimate through SNS technique. We could see that the density estimation is independent of temperature estimation but relies on probe power measurements.

In figure 2 we show SNS and its fitting with Lorentzian line as described above. Integral under the Lorentzian line shape,  $S_{at}\Delta\nu/2$ , is proportional to *N*, thus by fitting the noise spectrum we can acquire density of atomic ensemble. Most of the variables are calculable or measurable in fine accuracy, and SNS apparatus is easy to set up, so it's a relatively reliable and convenient method for vapor density calibration at both low and high temperature.



Figure. 2. spin noise spectroscopy: Dark blue line is measured spin noise from  $Rb^{87}$  vapor at 102 °C and under 834.15 nT magnetic field, yellow line is fitting of data with Lorentzian function. Acquisition time is  $\Box_{aqc} = 0.5$  s with sampling rate of 200 kHz.

At very high temperature, fast collision among atoms happens, that causes a large spin-exchange rate, Rse, bringing large spin relaxation. However when Lamor frequency induced by exerted magnetic field is much lower spin-exchange rate, ,i.e.,  $R_{se} \gg v_L$ , atoms in upper and lower hyperfine levels precess as a bond state. This gives a high signal to noise ratio and narrower linewidth, but with slower-down presession frequency. The precession frequency and magnetic resonance linewidth changes under different spin-exchange rate or precession rate can be illustrated as[9]:

$$\Delta\omega + i\omega_{q} = \frac{([l]^{2}+2)R_{se}}{3[l]^{2}} - \sqrt{-\omega_{0}^{2} - \frac{2i\omega_{0}R_{se}}{[l]} + \left(\frac{([l]^{2}+2)R_{se}}{3[l]^{2}}\right)^{2}}$$
7)

where  $[I]=2I+1\_, and I is nuclear spin, <math display="inline">\omega_0$  is the Lamour frequency of atoms. When  $R_{se}\gg\omega_0$ . the system enters SERF-regime, and the precession rate becomes  $\omega_q=\omega_0/q_1$ , with slowing down factor  $q_l.$ 

SNS can also be used as a convenient indicator for atomic ensemble entering spin-exchange relaxation free (SERF) regime, since it gives a magnetic resonance signal with a simpler setup comparing with a full magnetometer setup. In figure 3 we show theoretical prediction based on equation (7) and also experimental results of  $\omega_q$  from SNS measurements at 176 C. We could see the experimental results agree with theory nicely and support for a SERF-regime calibration. Besides, by fitting the experimental data, we could get a more precise density calibration at very high temperature.



Figure. 3. frequency shift in SERF regime: Blue line is the theoretical prediction of Lamour frequency based on equation (7) with fixed  $R_{se} = 4.24 * 10^5 s^{-1}$ . red dots are experimental result acquired at 176 °C.

(

## IV. COMPARISON OF DIFFERENT CALIBRATION METHODS

In figure 3 we show the density estimation of three different calibration methods. The black line denotes for estimated vapor density with thermodynamic function. Since the temperature sensor is set at the surface of vapor chamber, it estimates a higher temperature of the atomic vapor. This method should over estimate the atomic density. The purple line shows the density estimated by absorption spectrum. Due to strong absorption effect, and its reliance on temperature estimation it can not give a reliable estimation at high temperatures. Since the density estimation method based on SNS technique relies on the probe power measurements, we did the test under different probe power. As shown in figure 3, the blue, red, green lines stands for density estimation under 2 mW, 3 mW, and 4 mW probe power. We could see, they have good agreement under different power. This shows its robust density estimation ability. We could see at low temperature, the density estimation with different methods can easily be consist with each other, but at high temperature, the accurate density estimation is a crucial problem.



Figure. 3. comparison of different density estimation method from 60 °C to 110 °C.

## V. CONCLUSION

In conclusion, we proposed a more reliable atomic density estimation method at high temperature conditions, which is based on SNS technique. We have studied different methods for atomic density estimation, and compared their results in temperature range of 60  $^{\circ}$ C -

110 °C. We demonstrated the density estimation consistency by using SNS with different probe powers.

## REFERENCES

- [1] Budker, D., Romalis, M. Optical magnetometry. *Nature Phys* **3**, 227–234 (2007).
- [2] C. Troullinou, R. Jiménez-Martínez, J. Kong, V.G. Lucivero, and M. W. Mitchell Phys. Rev. Lett. 127, 193601
- [3] Andrew D. Ludlow, Martin M. Boyd, Jun Ye, E. Peik, and P. O. Schmidt Rev. Mod. Phys. 87, 637
- [4] Walker, T. G. / Larsen, M. S. Chapter Eight Spin-Exchange-Pumped NMR Gyroscope 2016 Advances In Atomic, Molecular, and Optical Physics Academic Press p. 373-401
- [5] Kong, J., Jiménez-Martí nez, R., Troullinou, C. et al. Measurement-induced, spatially-extended entanglement in a hot, strongly-interacting atomic system. Nat Commun 11, 2415 (2020)
- [6] Alcock, C. B. / Itkin, V. P. / Horrigan, M. K. Vapour Pressure Equations for the Metallic Elements: 298 – 2500K 1984-07
- [7] Weller, Lee / Bettles, Robert J. / Siddons, Paul / Adams, Charles S. / Hughes, Ifan G. Absolute absorption on the rubidium D1 line including resonant dipoleJournal of Physics B: Atomic, Molecular and Optical Physics, Vol. 44, No. 19 p. 195006
- [8] Crooker, S. A. / Rickel, D. G. / Balatsky, A. V. / Smith, D. L. Spectroscopy of spontaneous spin noise as a probe of spin dynamics and magnetic resonance 2004-09
- [9] Happer, W. / Tam, A. C. Effect of rapid spin exchange on the magnetic-resonance spectrum of alkali vapors *Phys. Rev. A*, Vol. 16 1977-11
- [10] Tycho Sleator, Erwin L. Hahn, Claude Hilbert, and John Clarke Phys. Rev. Lett. 55, 1742 - Published 21 October 1985
- [11] Rugar, D., Budakian, R., Mamin, H. et al. Single spin detection by magnetic resonance force microscopy. Nature 430, 329 - 332 (2004)
- [12] Budakian, R. / Mamin, H. J. / Chui, B. W. / Rugar, D. Creating Order from Random Fluctuations in Small Spin Ensembles 2005
- [13] Lucivero, Vito Giovanni / Jim é nez-Mart\inez, Ricardo / Kong, Jia / Mitchell, Morgan W. Squeezedlight spin noise spectroscopy 2016-05