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A calibration procedure for dc resistance ratio bridges

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Abstract – **Current comparator bridges are nowadays widespread for the realisation of the resistance unit with the quantum Hall effect and of a midrange resistance scale in most national metrology institutes. Future quantum resistance standards, e.g. those based on novel device materials and tabletop dry cryostats, make the more achievable DC currrent comparator bridges (DCCs) the most viable alternative with respect to the more accurate but more expensive cryogenic current comparator bridges (CCCs). A calibration of the individual DCC ratios of interest by comparison with a reference CCC is a straightforward way to improve the DCC performances. The paper reports a calibration exercise on the ratio** $12.9064\,\mathrm{k\Omega} : 1\,\mathrm{k\Omega}$**, chosen as a benchmark since** $12.9064\,\mathrm{k\Omega}$ **is close to the quantized resistance value. A more complete dataset involving also decadal resistance ratios will be presented at the Conference, together with a full evaluation of the calibration uncertainty.**

I. INTRODUCTION

Primary dc resistance metrology is based on the realisation of the resistance unit with the quantum Hall effect and of a resistance scale with dc resistance ratio bridges. For this purpose, in the midrange resistance scale (say, from $1\,\Omega$ to $100\,\mathrm{k\Omega}$) modern metrology laboratories employ current comparator resistance bridges.

In a current comparator [1] the currents I_1 and I_2 flow through windings having N_1 and N_2 turns, and generate two magnetomotive forces $N_1 I_1$ and $N_2 I_2$ in opposite directions. A null flux condition gives the measurement equation $N_1 I_1 = N_2 I_2$. In a resistance bridge, the currents I_1 and I_2 flow through the two resistors R_1 and R_2 under comparison with a voltage drop[1] $V = R_1 I_1 = R_2 I_2$.

Two major classes of current comparator bridges are available:

DCC bridges, DC current comprator resistance bridges. In these bridges the magnetic flux generated by the windings follows a path determined by a high-permeability ferromagnetic core and shields. The magnetic balance condition is sensed with a fluxgate technique [2]. DCC bridges can be operated at room temperature. Fully-automated commercial versions are available; the bridge ratios available do not typically extend much beyond the 0. to 10 range. The specified accuracy is of parts in 10^7 to 10^8.

CCC bridges, Cryogenic current comparator resistance bridges. [3]. The flux path is determined by the Meissner effect in superconducting shields; the magnetic balance is sensed by a superconducting quantum-interference device (SQUID) magnetometer. CCC bridges are typically semi-automated; commercial versions are available. They are more expensive than DCCs and the operating costs are boosted by the need of liquid helium supply to achieve the cryogenic temperatures, and well-trained operators. The base accuracy reaches a few parts in 10^9.

Traditional QHE experiments, performed with GaAs devices, require large cryomagnets with magnetic fields, e.g. in the order of $10\,\mathrm{T}$, and temperatures around $1\,\mathrm{K}$, operated with large amounts of liquid helium. The financial and technical expenditures related to the operation of a CCC are considered minor with respect to the operation of the QHE resistance standard itself.

In the last few years, however, QHE metrology research focused on graphene devices, which can operate at higher temperatures and lower magnetic fields [4]; high-accuracy QHE experiments in tabletop dry cryostats have been performed [5]. Very recently, measurements of the quantum anomalous Hall effect [6] in topological insulators show that an accurate quantum resistance standard can be achieved [7] with a small permanent magnet. Such developments forecast future low-cost, easy-to-operate quantum resistance standards in compact dry cryostat, available also to smaller national metrology institutes, calibration centers and industry. In this new framework, operating a CCC might become a major bottleneck. DCCs represent thus the most viable alternative.

DCCs are less performant in terms of accuracy than CCCs. A calibration of the DCC bridge ratio readings using the more accurate CCC as a reference ratio standard is the most straightforward way to improve the DCC performances.

This paper reports a calibration exercise of two widespread models of a commercial DCC bridge by comparison with a reference CCC bridge. The calibration

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[1] Actual bridges working conditions can be *close* to equilibrium; the small deviations are taken into account in the measurement model.

method proceeds by measuring the value of the resistance ratio between two thermostated resistors with the DCC and with the CCC, with the same measurement currents, and in short temporal sequence. The method has been considered in literature [5, 8]; it is here analyzed in detail.

The calibration determines the DCC bridge ratio error with an uncertainty in the order of 1×10^{-8}. The known error value can be employed to correct the DCC readings when employed for the realisation of the resistance scale or for calibration for customers, improving the measurement reliability and uncertainty with respect to the case when the sole DCC bridge specifications are employed. The results here reported focus on the specific ratio $12.9064\,\mathrm{k\Omega} : 1\,\mathrm{k\Omega}$, which has been chosen as a benchmark[2]: since $12.9064\,\mathrm{k\Omega}$ is approximately the value of the quantized Hall resistance in GaAs or graphene devices, the ratio is the first step in the realisation of a resistance scale.

The measurement exercise is ongoing at the present time. A more complete dataset involving also decadal resistance ratios will be presented at the Conference.

II. INSTRUMENTS AND STANDARDS

A. *CCC bridge*

The CCC bridge employed is manufactured by Magnicon GmbH on design of the Physikalisch-Technische Bundesanstalt (PTB). The windings turn numbers are selected by manually composing them by connecting in series individual windings having turn numbers in an (approximate) binary sequence; up to about 4646 turns can be achieved. Fractional turn numbers are simulated by a compensation network. After this initial manual setup, the measurement process is automated. The base ratio accuracy is in the 10^{-9} range.

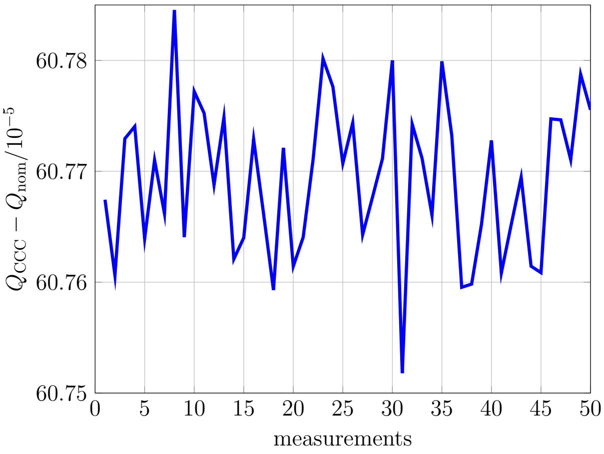
B. *DCC bridge*

Two DCC bridges, a Measurement International MI6010B purchased in 2006 and MI6010D purchased in 2021, have been employed in the calibration exercise. These models are widespread in calibration laboratories[3]. The specified measurement range of the MI6010B is $1\,\mathrm{m\Omega}$ to $13\,\mathrm{k\Omega}$ with a base accuracy of 5×10^{-8}. The specified measurement range of the MI6010D is $1\,\mathrm{m\Omega}$ to $100\,\mathrm{k\Omega}$ with a base accuracy of 3×10^{-8}.

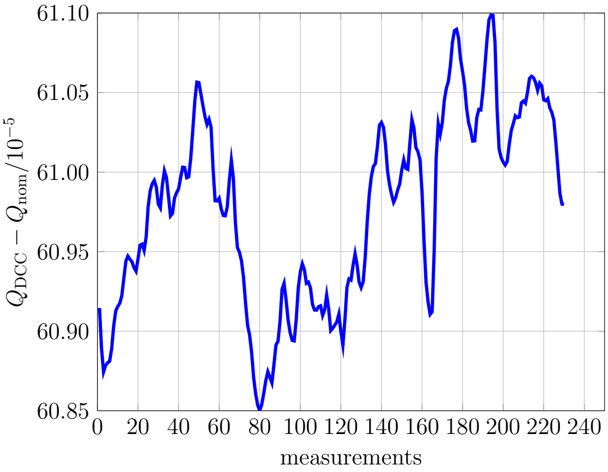
C. *Resistance standards*

The standard resistors employed were:

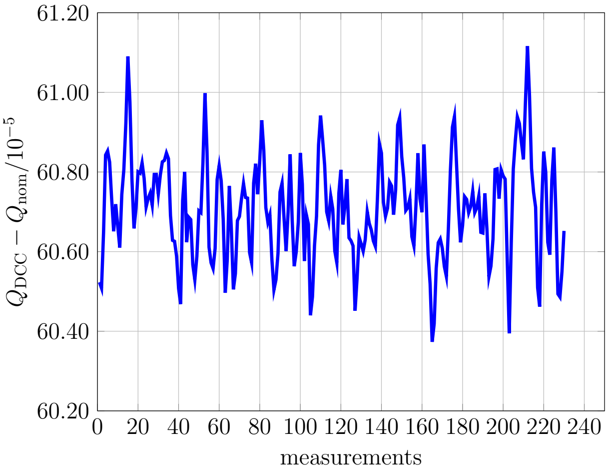
- STR1, a tailored resistor with a nominal value of $12.9064\,\mathrm{k\Omega}$, constructed from an Electro Scientific Industries (ESI), now IET Labs, model ESI SP5120.



(a) CCC



(b) MI6010D



(c) MI6010B

Fig. 1. Time series of the readings given by the three instruments involved in the calibration exercise.

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[2] Because of the turn numbers available in the specific DCCs selected for the exercise, the ratio $12.9064\,\mathrm{k\Omega} : 10\,\mathrm{k\Omega}$ has a larger uncertainty.

[3] A higher-accuracy model, MI6010Q, is available from the same manufacturer

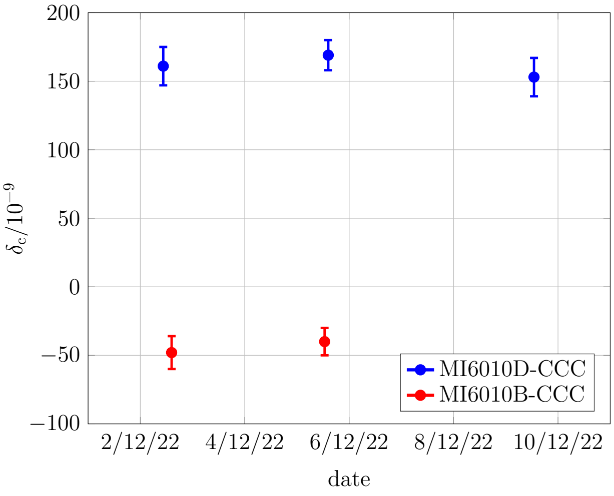

Fig. 2. Outcome of the calibration. δ_c is the relative difference between the DCC and the CCC measurement, versus the measurement date. The uncertainty bars display the standard uncertainty ($k = 1$), see also Table I.

The resistor is enclosed in a thermostatic air bath with a nominal temperature of 27 °C with a stability within a few mK.

- STD VH01, a custom-made resistor with a nominal value of $1\,\mathrm{k\Omega}$, made from the parallel of 10 Vishay VHA 512T $10\,\mathrm{k\Omega}$ components (tolerance $\pm 0.005\%$) thermostated at about 27 °C.

III. EXPERIMENTAL

The calibration is performed by measuring in short temporal sequence the resistance ratio with the DCCs and with the CCC, which acts as reference ratio standard. The calibration value δ_c is the deviation

$$\delta_c = \frac{Q_{\mathrm{DCC}} - Q_{\mathrm{CCC}}}{Q_{\mathrm{nom}}} \tag{1}$$

of Q_{DCC}, the ratio reading from the DCC bridge, from Q_{CCC}, the corresponding reading from the CCC bridge, relative to the nominal ratio Q_{nom}.

IV. UNCERTAINTY

The contributions to the calibration uncertainty are the uncertainty reference ratio measurement provided by the CCC, the statistical uncertainty of the readings of the DCC in the course of the calibration event, and the uncertainty associated to the stability and definability of the resistors employed as transfer standards.

A. *CCC uncertainty*

The uncertainty of the CCC is under evaluation. Provisionally, an uncertainty of 2×10^{-9} has been assigned to the measurements.

B. *DCC Type A uncertainty*

The uncertainty of the comparison is dominated by the Type A uncertainty of the DCC bridge. Since the readings time series shows a significant degree of autocorrelation, the approach proposed in [9] has been followed. Details are given in [10].

C. *Resistance standards*

The stability of the resistors during the comparison (for which a duration $\Delta t = 6\,\mathrm{h}$ is considered) is a source of measurement uncertainty. In particular, the effects to be considered are the temperature, the pressure and time stability. The resistance standards involved in the reported measurements are stable in the Δt considered, as reported in Section ii.C., and this uncertainty component is thus negligible.

V. RESULTS

Figure 1 gives the time series of measurements performed with the three instruments involved in the comparison.

Figure 2 shows the measured deviations δ_c for repeated measurements.

Table 1 gives a preliminary uncertainty budget for δ_c, see Sec. iv.

VI. CONCLUSIONS AND OUTLOOK

The calibration exercise shows that it is possible to calibrate the ratio error of commercial DCC bridges by comparison with a CCC. That the measured errors have a good reproducibility over the measurement period, which encourages the possibility to performing in-use corrections to the DCC readings on the basis of the calibrated values. The calibration exercise is being extended to more values, in particular to those required to use the DCCs in the realisation of a primary resistance scale, covering ratios between decadic resistance values ($10\,\mathrm{k\Omega}$, $1\,\mathrm{k\Omega}$, $100\,\Omega$, 10 ohm and $1\,\Omega$). These results, together with a full evaluation of the CCC measurement uncertainty, will be shown and discussed at the Conference.

VII. ACKNOWLEDGMENT

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VIII. *

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	CCC		DCC	
$\delta_\mathrm{c}\,/10^{-9}$	Type A $/10^{-9}$	Comb $/10^{-9}$	Type A $/10^{-9}$	$u(\delta_\mathrm{c})\,/10^{-9}$
MI6010D				
161	0.8	1.3	14.2	14
169	0.7	1.4	11	11
153	0.6	1.3	14	14
MI6010B				
-48	0.8	1.3	12.3	12
-40	0.7	1.4	9.8	10

Table 1. Uncertainty budget for the calibrated value δ_c *reported in Fig. 2.*

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