Multifunctional Hollow Dielectric Resonator Design for Conductivity/Permittivity Measurements of Bulk Samples

Kostiantyn Torokhtii[1,2], Nicola Pompeo[1], and Enrico Silva[1]

[1]*Università Roma Tre, Dipartimento di Ingegneria, Via Vito Volterra 62, 00146, Rome, Italy*
[2] *kostiantyn.torokhtii@uniroma3.it*

Abstract − **A hollow dielectric resonator has been developed for the measurement ofeither the conductivity or the permittivity ofbulk samples in form of small cylinders. The setup could be used to measure the dielectric permittivity of low-volume liquid samples. Open and closed (Hakki-Coleman) configurations are presented.**

Keywords − **Hollow dielectric resonator, Microwaves, Volume perturbation, Conductivity, Permittivity.**

I. INTRODUCTION

The characterization of the electromagnetic properties of a given material at the development stageof new devices or techniques is a main need in laboratory practice as well as in production processes. The wide implementation of new high-frequency technology often requires the characterization of materials at microwave frequencies [1]. As measurement devices for material characterization, resonators [2] are preferable over non-resonant methods like coaxial probes [3, 4] thanks to their high sensitivity and accuracy. Moreover, non-destructive characterization of materials is often required. This is extremely important when a series of completely different measurementsis needed. In addition, it is often found that only limited size samples can be produced or obtained, so that the characterization process must be designed to cope with this constraint.

In this paper, we propose to use a hollowcylindrical dielectric resonator (HCDR) tomeasurethe surface impedance (Z_s) of conducting bulk samples (e.g. single crystals), small cables or cable strands [5]. The device we exploreis conceived also to be used for the characterization of dielectric properties (real permittivity and loss tangent) of low-volume samples, also in the liquid phase.

II. MEASURED QUANTITIES AND HCDR PARAMETERS

Ahollow dielectric resonator is based on a conducting cavity (electromagnetic shield) loaded with a hollowdielectric rod, in the shape of a cylindrical tube, whose central empty volume accommodates small size conductors, dielectrics or liquid-filled vessels (Fig.1).

The sample placed in the hole of the dielectric rod perturbs theresonator electromagnetic field,thus giving rise to a resonator response related to the conductivity or permittivity of the sample itself.This structure has beenrecently usedfor the measurements of conductivity of superconductors [6], for the dielectric characterization of lossy liquids [7] and for the determination of the permeability of ferrites [8].

In the presently discussed volume perturbation technique, the surface impedance $Z_s = R_s + iX_s$ of a bulk conductive sample can be obtained by measuring the change of the HCDR Q-factorand resonant frequency f_{res}forthe mode of operation chosen. By performing two
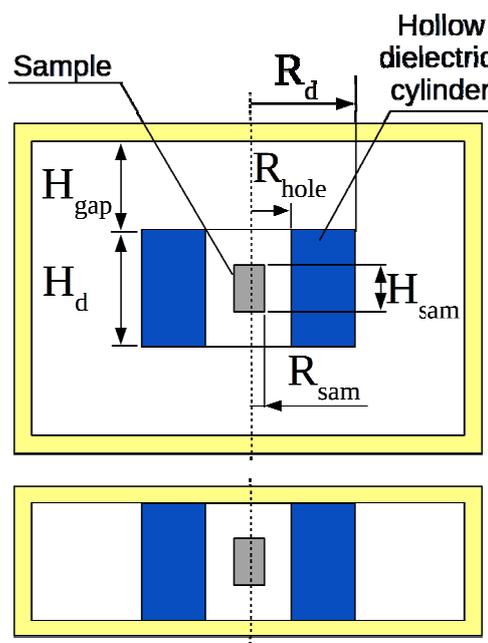

Fig. 1. Sketch of the hollow cylindrical resonator with its shield. Top, gapped configuration. Bottom, Hakki-Coleman configuration. The relevant geometrical dimensions are sketched. The sample location is depicted.

measurements with (f_{res}, Q) and without the sample ($f_{res,0}$, Q_0), one can obtain the real and imaginary parts of Z_s using the following relations [6]:

$$R_s = G_s \left[\frac{1}{Q} - \frac{1}{Q_0}\right] \qquad (1)$$

$$\Delta X_s = -2G_s \frac{\Delta f_{res} - \Delta f_{res,0}}{f_{res,0}} \qquad (2)$$

Here, Δf_{res} is the variation of resonant frequency with respect to some reference value. G_s is a constant geometrical factor which can be calculated using analytic relations, when available, or by finite elements simulations. It should be noted that for normal conductors $R_s = X_s$ and Eq. (1) can be used to yield the reference value for ΔX_s in Eq. (2), allowing to determine the absolute X_s.

When a dielectric (also in the liquid phase) sample with complex permittivity $\varepsilon = \varepsilon_0 \varepsilon'(1 + i\tan\delta)$ is placed inside the resonator, on the other hand, the following equations hold:

$$\tan\delta = \frac{1}{\eta}\left[\frac{1}{Q} - \frac{1}{Q_0}\right] \qquad (3)$$

$$\frac{\Delta \varepsilon'}{\varepsilon'} = -\frac{2}{\eta}\frac{\Delta f_{res} - \Delta f_{res,0}}{f_{res,0}} \qquad (4)$$

where η is a geometrical filling factor, computed for the volume occupied by the dielectric under study.

III. BASIC DESIGN OF HCDR

The main design requirements for the HCDR are (*i*) the sensitivity to the electromagnetic properties (conductivity, complex permittivity) of the sample to be placed in the of the HCDR and (*ii*) an accuracy as large as possible. First of all, we fixed the HCDR band of operation to be accessed by low-cost Vector Network Analysers (VNA) (typically limited to a maximum frequency equal to 15 GHz): hence, an operating frequency $f_{res,0} \sim 10$ GHz was chosen.

In order to maximize the device sensitivity, a large Q-factor is required. The main source oflosses in the DR is the ohmic dissipation in its conducting parts. The latter can be minimized by choosing high conductivity materials for the cylindrical cavity. Copperis the typical choice, with a surface resistance $R_s < 30$ mΩ at 10 GHz.

Another critical design step is the choice of the dielectric material for the dielectric rod. Indeed, it both concurs in defining the size of the HCDR for a given operating frequency (through the real part of its permittivity) as well as the sensitivity and accuracy (through its loss tangent, which has to be as low as possible). Among the most commonly used low-loss dielectric materials, TiO_2 (at low temperatures [9]), $LaAlO_3$ and Al_2O_3(sapphire), sapphire hasthe lowest $\tan\delta \sim 10^{-6}$(at room temperature). In addition, sapphire has a sufficiently high relative permittivity $\varepsilon' \sim 9$ (in-plane permittivity, sapphire is anisotropic)to allow reasonably compact HCDRs.

Taking into account the chosen resonant frequency range, we first considered the simpler Hakki-Coleman (H-C) geometry [10] (Fig.1, bottom panel). The H-C HCDR consists of a conducting cylindrical cavity coaxiallyloaded with a dielectric hollow cylindrical rod clamped between the bases. It is then possible to take advantage of the knowledge of the behaviour of simple bulk, cylindrical DRs.The usual TE_{011} mode has been chosen since it is well separated in frequency from spurious modes.

Using the electromagnetic model of H-C cylindrical DR as in[11, 12] we found that forcylinders with radius $R_d$and height H_d,an aspect ratioR_d/H_d in the range 0.66–0.86 yields the maximum frequency separation of the TE_{011} mode from othernearbymodes. Thus, $R_d/H_d = 0.76$ was selected for both HCDR variants here considered:(*i*) a closed, H-Cconfiguration without any gap between the sapphire rod and the bases of the Cu shield, and (*ii*) an open configuration with an adjustable gap between the dielectric rod and the conducting bases. In the latter casethe resonant frequency shifts due to the presence of thegap.

In order to obtain the desired operating frequency $f_{res,0} \sim 10$ GHz, a cylindrical HCDR with $R_d = 5$ mm and $H_d = 6.5$ mm was considered.

IV. HCDR GEOMETRY OPTIMIZATION

We designed the HCDR to be a general-purposedevice for the measurements of the properties of different types of samples such as small conducting wires, bulk/ single crystal conductors and dielectrics, and low-volume liquid dielectrics.In this communication we focus on the part of the design process concerning conducting samples. Our goal is to be able to measure samples withcylindrical shape withtypical radius$R_{sam} = 0.5$ mm and $H_{sam} = 2$ mm, to be placed in the center of the HCDR (see Fig.1).

The optimization of the HCDR has been performed using finite elements simulations.
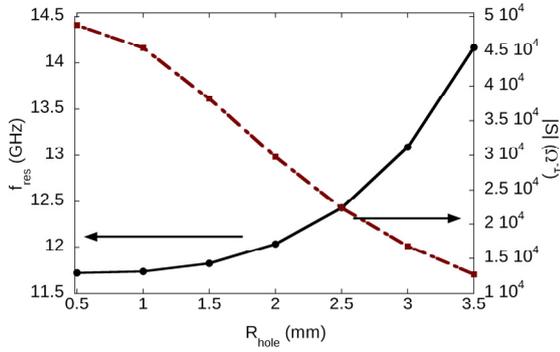
Fig. 2. Change of the TE_{011} mode resonant frequency and sensitivity of H-C HCDR with the same conducting sample of diameter of 1 mm.

Fig. 3. HCDR TE_{011} sensitivity S variation with respect to non-gapped HCDR (S_0) for different hole radii R_{hole}.

To determine the optimal hole radius R_{hole} we started from the H-C configuration. To allow sufficient room for the sample, that we take as a cylinder of maximum diameter of 1 mm, R_{hole} was varied from 0.5 mm (sample in contact with sapphire) to 3.5 mm (a sapphire wall of the HCDR thinner than 1.5 mm becomes unusable). Considering a conductive sample with typical conductivity $5.8 \cdot 10^7$ S/m, our finite-element simulations show that by increasing the hole radius, the Q-factor increases from 8000 to 10000, mainly because of the reduced effect of the sample losses. The conductive nature of the sample yields an electromagnetic field expulsion and thus an evident increase of the resonant frequency up to the frequency limit we set. It has to be stressed that, in this case, high Q does not directly imply high sensitivity, since G_s increases as well: indeed, one has for the sensitivity $S = -Q^2/G_s$. In Fig. 2 we show the variations of the calculated sensitivity and of the resonant frequency for the TE_{011} mode vs. R_{hole}. By varying R_{hole} we have then to choose a trade-off between acceptable frequency variation, which affects the surface reactance X_s, and a reduced sensitivity on R_s. An acceptable compromise is with R_{hole} = 2.25 mm.

Improvement of the sensitivity is possible by taking into account that the introduction of a gap between the dielectric rod and the conducting bases improves the Q-factor [12, 13, 14]. This effect can be used to circumvent the reduction of the sensitivity at larger R_{hole}. We then performed an analysis of this effect on the sensitivity of the HCDR, thus departing from the H-C configuration: by keeping fixed the metallic cavity radius, we added symmetrical air gaps between the dielectric rod and the cavity bases (Fig.1). In Fig. 3 we present the improvement of the sensitivity with the introduction of gaps with respect to non-gapped configuration for different R_{hole}. It is seen that the introduction of the air gaps determines a huge increase of the computed sensitivity. With relatively small gaps of ~ 2 mm, the sensitivity increases mor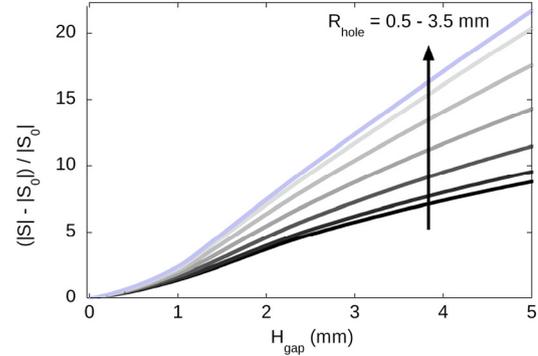e than a factor of 5. Moreover, it is found that larger hole diameters determine a larger gain in the HCDR sensitivity by increasing the air gaps. Finally, it can be shown that the gap decreases the TE_{011} mode frequency. This fact helps in ensuring that the operational frequency stays below 15 GHz.

V. CONCLUSIONS

We gave preliminary design recommendations for a general-purpose HCDR-based device for the measurement of the surface impedance of small bulk samples. The HCDR can be in principle used also for the measurement of the permittivity of dielectrics, also in the liquid phase. The inner hole diameter has been optimized mainly taking into account the sensitivity of the device. To improve the sensitivity, we studied the effect of additional gaps between the dielectric rod and the metal bases in terms of the HCDR sensitivity. The performed calculations have shown an improvement in the sensitivity up to 20-fold, with a 5 mm gap. A feasible gap of 2 mm with a dielectric rod hole of 2.25 mm yields an increase of sensitivity by a factor ~5 with respect to the closed (no gap) configuration. We conclude that the HCDR is a potential candidate for a sensitive measuring device for GHz properties of conductors or dielectrics.

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