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Use of high-resolution digital multimeters for the calibration of temperature sensors

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***Abstract*-**The calibration of precise negative temperature coefficient resistors (NTCRs), which have resistance $R_N = 10000\ \Omega$ at a temperature of 25 °C and limits of error ±0,2 °C in the range from 0 °C to 70 °C, by means of the reference platinum resistance thermometer Pt-25 (with the resistance of ≈25,5 Ω at 0 °C) in the range of (23 ± 4) °C, and the high-resolution digital multimeters (up to 8½ digits), is described. The influence of the resolution of the digital multimeter is investigated, as well as the influence of the self-heating of NTCRs and the measurement on different ranges and with different currents during the comparison of different type of sensors. The analysis of the resistance-versus-temperature (*R*-*T*) dependence is done by using the least-squares method and the two-parameter approximation curve for NTCRs. The results have shown that, in the range of (23 ± 4) °C, which is useful for the laboratory applications, NTCRs can be calibrated by means of Pt-25 thermometer and high-resolution digital multimeters with an uncertainty of a few millikelvin, which is very convenient for their wide application as precise and sensitive temperature sensors.

I. Introduction

NTCRs are passive semiconductor temperature sensors with very good sensitivity (≈5 %/K) and satisfactory long time stability. We can measure their resistance with a relatively small uncertainty by means of digital multimeters (even computer controlled), which makes NTCRs suitable for application in automated measurement methods. However, the most important thing concerning their use is to calculate the measured temperature as accurate as possible from the measured value of resistance.

In the Primary Electromagnetic Laboratory (PEL), which is a part of the Faculty of Electrical Engineering and Computing of the University of Zagreb, NTCRs of type UUA41J1, with resistance $R_N = 10000\ \Omega$ at a temperature $\vartheta = 25$ °C and limits of error ±0,2 °C in the range (0 to 70) °C are used [1], as well as of type UUA41J8 with the identical parameters except an extended temperature range (0 to 100) °C. To calculate the measured temperature from any measured value of resistance, an approximation curve is needed, but for its use a two parameters need to be determined for each NTCR. This can be done by the calibration of each NTCR, i.e. to measure its resistance at a known temperature. In such method, a known temperature can be obtained from the measurements made by other temperature sensor which is placed at the same temperature as calibrated NTCR, and as such "reference" temperature sensor another NTCR can be used, but also a platinum resistance thermometer Pt-25 [2] as well. Generally speaking, the platinum resistance thermometers are more accurate and represent the reference for the temperature measurements in the very wide temperature range. However, for the laboratory applications, a range of interest is around a reference temperature of 23 °C, and following these requirements the calibration range of interest is (23 ± 4) °C.

High-resolution (up to 8½ digits) digital multimeters (DMM) can be used for direct measurement of resistance of both type of temperature sensors (NTCRs and Pt-25). The calibration problem is two fold: (i) influence of the current flowing through a sensor, and (ii) different range of values. The NTCRs are sensitive to the self-heating, i.e. in order to achieve as accurate reading as possible, the measurement of resistance must be as short as possible (so called one-trigger-mode), and in the time interval to the next reading the current should not flows through the sensor; in contrary, usual use of Pt-25 requires that a constant current of 1 mA flowing through the thermometer. Furthermore, if we want to calibrate an NTCR by means of Pt-25, due to their different values of resistances for the same temperatures (at $\vartheta = 23$ °C their resistances are approx. 10920 Ω and 27,84 Ω, respectively), a DMM should measure at different ranges, and such situation could cause an additional errors in the results.

In this paper the correct measurement and experimental set-up for the calibration of NTCRs by means of Pt-25 thermometer with an uncertainty of a few millikelvin is presented, which is not an easy task.

II. Measurement set-up

A. Two-parameter approximation curve for NTCRs

The *R*-*T* dependence for an NTCR is described by the basic exponential equation:

$$R_T = R_{T_0}\exp\left[-\frac{B\vartheta}{T_0^2(1+\vartheta/T_0)}\right], \tag{1}$$

where table data indicate the value $R_{T_0} = 32650\,\Omega$ at temperature $T_0 = 273{,}15$ K; B is coefficient (expressed in kelvin) and $\vartheta = T - T_0$ (i.e. Celsius temperature). Replacing the coefficient B in (1) by the parabolic function $B = B_0(1 + b\vartheta - c\vartheta^2)$, and after simplification and neglecting of the higher order factors, (1) becomes:

$$\ln(R_T/R_{T_0}) = -C_1\vartheta(1 + C_2\vartheta)^{-1}, \tag{2}$$

where

$$C_1 = B_0/T_0^2, \qquad C_2 = (T_0)^{-1} - b\,. \tag{3}$$

This is so-called two-parameter approximation curve for the calculation of the measured temperature T from the measured resistance R_T. More details about the derivation, use and comparison with some other approximation curves are given in [3-5].

Parameters C_1 and C_2 have to be determined for each NTCR if we want to apply two-parameter approximation curve (2). Basically, they can be calculated according to the least-squares method [6-9], using the measured resistance R_T at the known temperature T, within the temperature range of interest, by rearranging (2) as:

$$C_2\vartheta\ln(R_T/R_{T_0}) + C_1\vartheta = -\ln(R_T/R_{T_0})\,. \tag{4}$$

With the substitutes $m = \vartheta\ln(R_T/R_{T_0})$, $n = \vartheta$ and $f = -\ln(R_T/R_{T_0})$, the following system will be obtained:

$$\begin{aligned}[mm]C_2 + [mn]C_1 &= [mf]\\ [mn]C_2 + [nn]C_1 &= [nf]\,,\end{aligned} \tag{5}$$

where Gauss's notation is used for the sums: for instance, $[mn] = \sum_{i=1}^{r}\vartheta_i^2\ln(R_{Ti}/R_{T_0})$, $[nn] = \sum_{i=1}^{r}\vartheta_i^2$, etc., where r is equal to the number of pairs $R_{Ti} \leftrightarrow \vartheta_i$ used in the calculation, and for temperature range 19 °C to 27 °C, with the step of approx. each 0,5 °C, $r = 17$. Determination of these parameters for used NTCRs is given in section III.

B. Calculation of the temperature from the platinum resistance thermometer Pt-25

The calculation of the temperature T from measured resistance $R(T)$ of the platinum resistance thermometer Pt-25, for the temperature range $0{,}01\ °\mathrm{C} \le T \le 29{,}7646\ °\mathrm{C}$ is based on the reference [10]; for the simplification, a common index "90" is omitted from the equations. The constants of particular Pt-25 thermometer needed in calculations are $R(0{,}01\ °\mathrm{C})$ and a. As a first step we are calculating the ratios

$$W(T) = \frac{R(T)}{R(0{,}01\,°\mathrm{C})}, \tag{6}$$

$$W_\mathrm{r}(T) = W(T)[1 - a] + a\,. \tag{7}$$

In the final step, a Celsius temperature is calculated as following:

$$\vartheta/°\mathrm{C} = D_0 + \sum_{i=1}^{9} D_i\left[\frac{W_\mathrm{r}(T) - 2{,}64}{1{,}64}\right]^i; \tag{8}$$

the values of constants are given in Table I.

Table I. The constants D_0 to D_9 for the calculation of temperature according to (8), and constants of used Pt-25

D_0	439,932 854	D_5	0,005 184	$R(0{,}01\ °\mathrm{C})$	25,5064 Ω
D_1	472,418 020	D_6	–0,963 864	a	–0,00025104
D_2	37,684 494	D_7	–0,188 732		
D_3	7,472 018	D_8	0,191 203		
D_4	2,920 828	D_9	0,049 025		

C. Experimental set-up

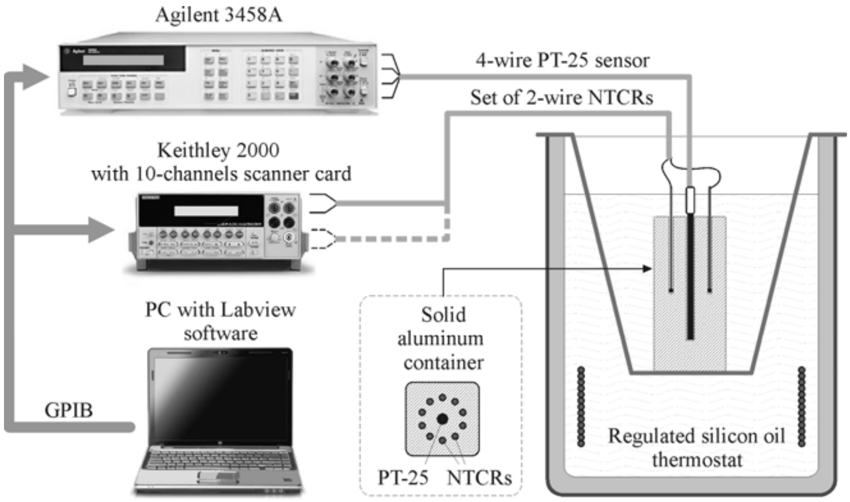

Fig. 1. The set-up scheme for calibration of NTCRs by means of Pt-25 thermometer and two PC-controlled digital multimeters

The scheme of experimental set-up for calibration of NTCRs by means of thermometer Pt-25 and high-resolution digital multimeters is shown in Fig. 1. The major requirement for correct calibration of resistance thermometers is attaining of equal thermal distribution along compared sensors at certain temperature point. That so-called "thermal short-circuit" has been realized by placing the set of NTCRs together with Pt-25 in the solid aluminium container of rectangular prism shape. The container has one hole in the middle for insertion of Pt-25, and ten more deep holes drilled concentrically around the centre hole for insertion of NTCRs. Such composition of "thermally short-circuited" resistance thermometers is immersed into laboratory thermostat filled with silicon oil, which temperature can be regulated by electric heater.

The calibration of resistance thermometers started by cooling the thermostat in refrigerator to the temperature of approx. 15 °C, and afterwards the heating has been applied in order to obtain linear temperature increase of silicon oil thermal bath. To ensure proper thermal equalization of temperature across aluminium container, the calibration temperature range of 19 °C to 27 °C was covered with very slow heating process (duration of altogether 30 h). The scanning of resistances of compared NTCRs and Pt-25 has been performed every five minutes using two digital multimeters, simultaneously triggered via GPIB bus, and self-developed software written in LabVIEW environment. Resistances of NTCRs have been measured in fast sequence with 10-channel scanner card inserted into digital multimeter Keithley 2000 with 6½ digits resolution, set-up at 100 kΩ range. To avoid self-heating of NTCRs with measuring current, the close-state time of scanner relays was shorted as much as possible, i.e. 100 ms for each channel. On the other hand, the sensitivity of Pt-25 sensor is of one order of magnitude lower than sensitivity of NTCR sensor and requires steady current flow of 1 mA. For the measurement of its resistance, a DMM must be set up at 100 Ω range, and in the first set-up the second digital multimeter Keithley 2000 was used (not presented in Fig. 1), but for the aiming uncertainty of calibration of a few millikelvin and due to its limited resolution, the results were not satisfying (Fig. 2). Instead in the final set-up, a digital multimeter Agilent 3458A with resolution of 8½ digits has been used (presented in Fig. 1) on 100 Ω range and constant OHMF (4-wire) measuring mode which ensures steady current flow of 1 mA through Pt-25 sensor between readings; the overall results were at the required level of uncertainty (Fig. 3).

III. Results of calibration

For this calibration the experimental set-up is used as explained in the previous section. The Pt-25 thermometer is used as the reference one, by which the reference value of temperature ϑ_{ref} at measurement points (each 0,5 °C in the range 19 °C to 27 °C) is calculated, as explained in section II.B. Based on these data, from the measured resistances R_T of each NTCRs, their associated parameters C_1 and C_2 are calculated in the way explained in section II.A, from which one can calculate the measured temperature ϑ_{cal} using the equation:

$$\vartheta_{\text{cal}} = \frac{\ln(R_{T_0}/R_{\text{T}})}{C_1 - C_2 \ln(R_{T_0}/R_{\text{T}})} \; . \tag{9}$$

Important information about the performed calibration of NTCRs is the difference $\vartheta_{\text{cal}} - \vartheta_{\text{ref}}$, where ϑ_{cal} is calculated by (9) for particular NTCR in the temperature range of interest, while the reference temperature ϑ_{ref} measured by Pt-25 is calculated by (8). Results of such measurements for the NTCRs No. 3, 4, 8, 18 and 19 (internal marks), obtained in the set-up where resistance of Pt-25 is measured by DMM with 6½ digits resolution, are shown in Fig. 2. Instead of expected differences $\vartheta_{cal} - \vartheta_{ref}$ of a few millikelvin, the presented results show differences of one order of magnitude higher (which is unacceptable), but very similar for all NTCRs. Obviously, this is a systematic effect due to the limited resolution of the used DMM, and a sign that experimental set-up needs to be improved.

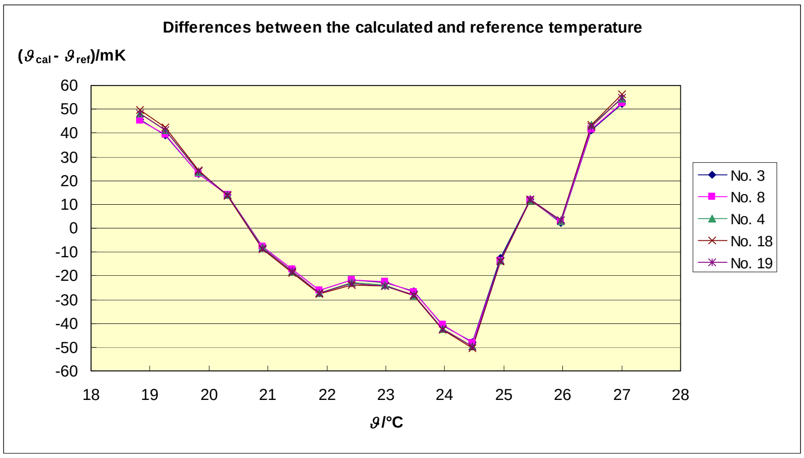

Fig. 2. Differences between the temperature calculated by the NTCRs and the reference temperature measured by Pt-25 by DMM with 6½ digits resolution – unacceptably large for the application of interest

The experimental set-up was changed and 8½ digits resolution DMM was used for measurement of resistance of Pt-25. As an example for this measurement, the raw measured data of resistance of NTCRs No. 2, 3, 18, 19 and Pt-25, as well as calculation of reference temperature ϑ_{ref} according to (8), are presented in Table II.

Table II. Measured resistances for NTCRs 2, 3, 18 and 19, and reference Pt-25 sensor in the range of (23 ± 4) °C

	2	3	18	19	Pt-25			
r	R / Ω	R / Ω	R / Ω	R / Ω	R / Ω	$W(t_{90})$	$W_r(t_{90})$	ϑ_{ref} /°C
1	13161,84	13163,21	13123,52	13132,63	27,42306	1,075144	1,075163	18,9094
2	12812,90	12814,17	12775,93	12785,03	27,48291	1,077491	1,077510	19,5013
3	12510,12	12511,67	12474,60	12483,67	27,53609	1,079576	1,079596	20,0273
4	12237,50	12239,15	12203,31	12212,36	27,58550	1,081513	1,081533	20,5162
5	11993,94	11995,57	11960,83	11969,84	27,63029	1,083269	1,083290	20,9594
6	11725,28	11726,74	11693,17	11702,03	27,68185	1,085290	1,085312	21,4696
7	11445,91	11447,37	11414,95	11423,89	27,73585	1,087407	1,087429	22,0041
8	11172,06	11173,48	11142,39	11150,95	27,79057	1,089553	1,089575	22,5458
9	10915,38	10916,82	10886,76	10895,17	27,84351	1,091628	1,091651	23,0699
10	10705,90	10707,38	10678,12	10686,64	27,88760	1,093357	1,093380	23,5065
11	10450,49	10451,74	10423,64	10432,11	27,94268	1,095516	1,095540	24,0520
12	10237,94	10239,22	10212,07	10220,38	27,98970	1,097360	1,097384	24,5178
13	10025,81	10027,14	10000,54	10008,89	28,03729	1,099226	1,099251	24,9892
14	9810,08	9811,48	9785,85	9794,13	28,08746	1,101193	1,101218	25,4863
15	9592,16	9593,60	9568,74	9577,20	28,13962	1,103238	1,103264	26,0032
16	9384,75	9386,12	9362,24	9370,21	28,18981	1,105205	1,105232	26,5007
17	9166,69	9167,96	9144,86	9152,80	28,24447	1,107348	1,107375	27,0425

As an example of the calculation of parameters for used NTCRs, the results for No. 3 from Table II are analysed, i.e. for $r = 17$ of pairs $R_{Ti} \leftrightarrow \vartheta_i$, and given in Table III. The parameters C_1 and C_2 are determined by calculating the needed sums from [mm] to [nf] and solving the system (5). Since the most interesting temperature is 23 °C, for that temperature the resistance is expressed (R_{23}), as well as derivation at this point, calculated as follows:

$$\mathrm{d}R_{23}/\mathrm{d}\vartheta_{23} = -C_1 R_T (1 + C_2 \vartheta)^{-2} . \qquad (10)$$

The last parameter s_ϑ is standard deviation of the calculated differences $\vartheta_{cal} - \vartheta_{ref}$.

Table III. Complete analysis for NTCR No. 3 (as an example) for the results presented in Table II

r	ln R_T/R_{T0}	*mm*	*mn*	*mf*	$(\vartheta_{cal}-\vartheta_{ref})$/mK
1	–0,90842	295,07	–324,82	–15,60	–1,03
2	–0,93529	332,68	–355,69	–17,06	–1,22
3	–0,95918	369,02	–384,72	–18,43	0,41
4	–0,98121	405,24	–413,00	–19,75	–0,61
5	–1,00131	440,44	–439,87	–21,01	2,65
6	–1,02397	483,31	–471,99	–22,51	–2,88
7	–1,04808	531,86	–507,46	–24,17	1,12
8	–1,07230	584,47	–545,06	–25,92	1,89
9	–1,09554	638,78	–583,07	–27,69	–0,19
10	–1,11491	686,84	–616,05	–29,22	–0,42
11	–1,13908	750,60	–658,95	–31,21	–0,10
12	–1,15962	808,34	–697,07	–32,97	–0,53
13	–1,18055	870,31	–737,21	–34,83	3,32
14	–1,20229	938,93	–780,95	–36,84	1,31
15	–1,22475	1014,26	–828,13	–39,01	–2,80
16	–1,24661	1091,38	–875,48	–41,18	0,40
17	–1,27013	1179,75	–928,84	–43,63	–1,40

Sums	
[*mm*] / K^2	11421,2791
[*mn*] / K^2	–10148,3816
[*mf*] / K	–481,0289
[*nn*] / K^2	9100,7601
[*nf*] / K	431,6506

Parameters	
C_1 / K^{-1}	0,050737777
C_2 / K^{-1}	0,002966166
R_{23} / Ω	10950,710
$(dR_{23}/d\vartheta_{23})/(\Omega/K)$	–486,912
s_ϑ / mK	1,70

* These parameters are pointed out also in Table IV

In Fig. 3 are presented results for the calibration of NTCRs No. 2, 3, 18 and 19 from the data given in Table II, and calculated in the same way as previously described for No. 3 and presented in Table III. It is obvious that the differences $\vartheta_{cal}-\vartheta_{ref}$ are within ±4 mK, which is very satisfactory and useful for intended use of NTCRs.

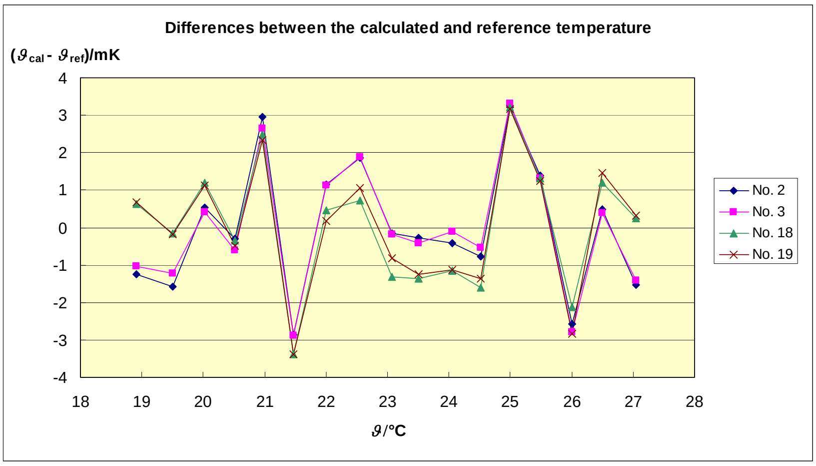

Fig. 3. Differences between the temperatures calculated by the NTCRs, based on the determined parameters C_1 and C_2, and the reference temperature measured by Pt-25 with the DMM with 8½ digits resolution

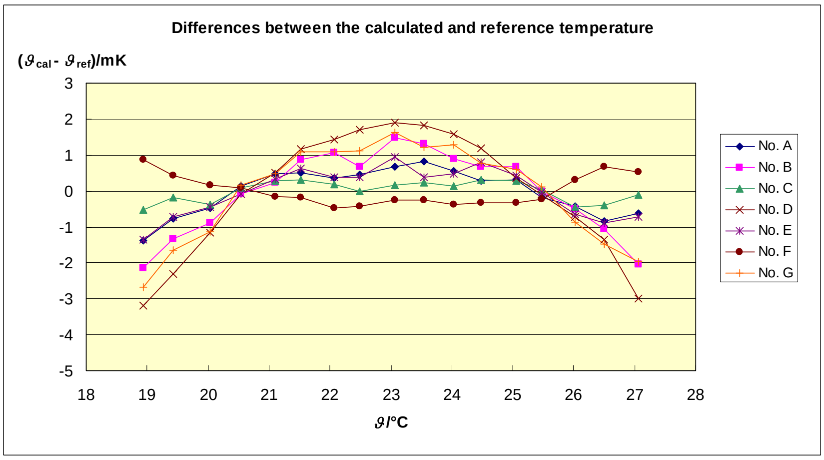

Fig. 4. Differences between the temperatures calculated by the NTCRs, based on the determined parameters C_1 and C_2, and the reference temperature measured by reference NTCR No. 18 and 19

We can use calibrated NTCR as the reference one for the calibration of some other NTCRs, using similar procedure but with only one DMM with 6½ digits resolution, since in that case all sensors have similar resistances; typical results are given in Fig. 4. In Table IV the summarised data of calibrated NTCRs are presented, and are calculated as described in the example for No. 3. The reference one (No. 2, 3, 18 and 19) were calibrated using the thermometer Pt-25, while other NTCRs are calibrated using some of reference NTCRs.

Table IV. Summarised data of calibrated NTCRs, determined as presented in Table III for No. 3

NTC No.	C_1 / K^{-1}	C_2 / K^{-1}	R_{23} / Ω	$(\mathrm{d}R_{23}/\mathrm{d}\vartheta_{23})$ / (Ω/K)	s_ϑ / mK
1	0,050472914	0,002850790	10983,572	−488,247	1,55
2	0,050745565	0,002967770	10949,287	−486,890	1,74
3	0,050737777	0,002966166	10950,710	−486,912	1,70
4	0,050960629	0,003121236	10938,157	−485,245	0,50
5	0,050594946	0,002904204	10968,464	−487,629	1,38
6	0,050544991	0,002844778	10964,945	−488,242	1,47
8	0,050634100	0,002898172	10957,653	−487,652	1,74
10	0,050712420	0,002967477	10957,028	−486,922	0,95
11	0,050368665	0,002813042	10998,560	−488,699	1,65
12	0,050940275	0,003062090	10927,744	−485,822	0,43
13	0,050963245	0,003105572	10933,522	−485,390	0,37
18	0,051070000	0,003150279	10919,966	−484,873	1,69
19	0,051020921	0,003139173	10928,617	−485,022	1,72
29	0,051016453	0,003076272	10913,515	−485,619	0,95
30	0,050588216	0,002859727	10958,578	−488,060	1,21
A	0,050771209	0,003016573	10955,806	−486,404	0,64
B	0,050660262	0,002915323	10955,897	−487,465	1,14
C	0,050924855	0,003088884	10938,250	−485,582	0,29
D	0,050440323	0,002809751	10980,697	−488,669	1,69
E	0,050754289	0,003007776	10957,530	−486,502	0,67
F	0,051165960	0,003236650	10919,648	−483,975	0,42
G	0,050488786	0,002863415	10983,070	−488,112	1,34
H	0,050307794	0,002738673	10993,790	−489,469	1,96
I	0,050810273	0,003019344	10947,318	−486,343	0,71
J	0,050235123	0,002711866	11004,146	−489,790	2,16
K	0,050489944	0,002886208	10988,680	−487,893	1,40
Mean	0,050708459	0,002962354	10956,737	−486,978	1,21
s	0,000250246	0,000134951	25,474	1,503	0,56

IV. Conclusions

The presented results show that, with careful experimental set-up and by using 8½ digits resolution DMM, NTCRs can be calibrated by means of thermometer Pt-25 with an uncertainty of a few millikelvin in the range of (23 ± 4) °C, which is very important for their use in laboratory applications for temperature measurements.

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