

# Digital Power and Energy Measurement

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**Abstract-** A brief description of the designed digital sampling electricity meter based on the modern Texas Instruments TMS320C6711 DSP is given. The electricity meter measures basic electric network parameters such as rms values of voltages and currents, powers, energy, power factor and net frequency. A brief discussion of measurement accuracy and error correction is also given. The designed calibration procedure of the current and parasitic phase shift between the measured voltage and current based on the active power measurement is described.

## I. Introduction

Basic instruments for the most accurate measurement of electric power and energy are digital electricity meters. They use digital multiplication of voltage and current samples, [1], [2], received from one A/D converter with multiplexed inputs, [3], or they may use separate A/D converter for each input signal, [4], [5]. The number of A/D converters depends on the desired accuracy and on the allowed price of the instrument. The advantages of such instruments are obvious: high accuracy, short- and long-term stability, complex net parameters measurements, possibility of remote automated data processing, auto-calibration, self-test and many other functions resulting from the microprocessor-based digital system possibilities.

Digital power and energy measurement is based on sampling and digitising of the instant values of the voltage and current in regular time intervals, their arithmetic multiplying in digital form and averaging of the products. Such power and energy measurement is influenced not only by the inaccuracies of analogue circuits but also by the inaccuracy of the sampling process itself, [6]. In the asynchronous sampling method the error can be higher and it needs longer summation time interval for the error to be sufficiently low. The synchronous sampling method gives better results. It requires shorter measuring time but its realisation is more complex.

Very important characteristic of electricity meters is their accuracy. It depends on the accuracy of the analogue input circuits, the accuracy of A/D conversion and the accuracy of digital calculations. In digital sampling electricity meters the measurement error can be simply eliminated in the digital signal processing. In this case the main problem is the stability of the parameters of these parts which handle the measured signals. The analogue input circuits must be constructed using highly stable components. There are many methods of error correction in digital electricity meters, [7]. Most of these methods use software correction based on calibration process. The instrument is calibrated using known values of the input quantity. During the calibration a digital processor in the instrument calculates correction coefficients or correction function which are used to get the correct values of the measured quantity during the measurement.

Digital electricity meters have different measurement and communication possibilities and different prices. The accuracy of the best instruments is of the order 0.01 %.

## II. Electricity meter brief description

The proposed instrument is able to measure all basic three-phase net parameters including rms values of voltages and currents, active, reactive and apparent power, power factor, net frequency, and energy delivered into the load. It can calculate the frequency spectra using FFT. The results can be transmitted via infrared or RS-232 interfaces. There is also a frequency output. Three-phase signal generator of voltages and currents is also combined in the electricity meter.

Block diagram of the proposed instrument for power and energy measurement (PEM 6711) is in Fig.1, [8]. Analogue part contains circuits for sensing and conditioning of the net voltages and currents. A block of six A/D converters digitises the signals from the analogue part with the sampling rate  $f_s = 50$  kHz. The DSP part with the Texas Instruments TMS320C6711 processor makes necessary calculations and signal processing in digital form. It also contains software for signal generation with the output through a D/A converters block and a frequency output with the frequency proportional to the measured power. To visualise the results, a communication port with the microcontroller was designed.

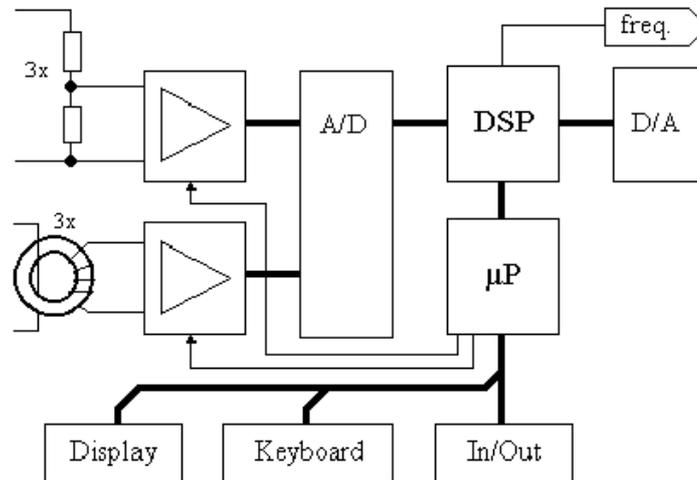


Figure 1. Block diagram of the designed electricity meter

Microcontroller board with the Texas Instruments TMS320F243 processor controls the operation of the device. It controls some circuits of the analogue part, displays the results and enables communication with other instruments via standard interfaces. For presentation of time and frequency characteristics, graphical LCD display with the resolution 240x128 dots is used.

### III. Current and phase shift calibration

One serious problem to be solved for accurate power calculation is the calibration process itself. In electricity meters, three calibration procedures must be run, [9]: voltage calibration, current calibration and parasitic phase shift correction. The best way is to carry out the calibration process automatically, using a computer. The computer controls a signal source (sets the desired measured values), reads the necessary values from the calibrated and reference instrument in the same instant, calculates the calibration constants and stores them in the memory of the calibrated instrument. Manufacturers of electricity meters usually have such possibility.

The designed instrument has an algorithm for voltage calibration using the least squares method. Current calibration can be done using the same algorithm but usually the measured current is not stable. In this case it is necessary to synchronize the measuring time period of the calibrated and the reference instrument or to use mean values. Another way is to use frequency output of the instrument with the output frequency proportional to the apparent or active power. The frequencies proportional to the measured powers are compared with the correct values in the precise reference comparator. In this way, the calculated apparent power,  $S_i$ , is compared with the correct power value  $S$ . As the voltage is correct (calibrated already) the mean value of the apparent power relative error is the same as the mean value of the current relative error. These errors in a few points over the calibrated current range can be used to calibrate the current by the least squares method.

Simple, lower accuracy electricity meters sometimes do not have the frequency output based on the apparent power. They have only the frequency output based on the active power. In this case such current calibration procedure must be repeated because of its dependence on the phase shift correction. A method was carried out to calculate the current and the phase shift at the same time.

The measured powers  $P_i$ ,  $P_{i\varphi}$  for two different phase shifts  $0$ ,  $\varphi$  are compared with the correct values  $P$ ,  $P_\varphi$ , respectively, using the frequency output of the instrument. The mean values of the relative errors yield the mean values of the calculated powers

$$P_i = P(1 + \frac{\delta}{100}), \quad P_{i\varphi} = P_\varphi(1 + \frac{\delta_\varphi}{100}) \quad (1)$$

The equations to be solved to get the mean values of the measured current and parasitic phase shift are, [9],

$$P_i = UI_i \cos(\varphi_p), \quad (2)$$

$$P_{i\varphi} = UI_i \cos(\varphi + \varphi_p), \quad (3)$$

where  $\varphi_p$  is a phase shift between the voltage and the current caused by the input circuits of the instrument and  $U, I_i$  are the measured voltage and current, respectively.

Using mathematical formula for the cosine of the sum of two phase angles, [10], and taking into account that  $\varphi_p$  is a small phase shift it is possible to simplify this equations. The simplest solution follows from the well known fact that if  $x$  is small then  $\sin(x) = x$  [rad] and  $\cos(x) = 1$

$$I_i = \frac{1}{U} \frac{2P_i^3 \sin^2(\varphi)}{2P_i^2 \sin^2(\varphi) - [P_i \cos(\varphi) - P_{i\varphi}]^2}, \quad (4)$$

$$\varphi_p = \frac{P_i \cos(\varphi) - P_{i\varphi}}{P_i \sin(\varphi)}. \quad (5)$$

For phase shifts  $\varphi_p$  up to  $5^\circ$  the error of this solution is of the order of tenth per cent what is not acceptable for precise electricity meters.

Higher precision of the solution is obtained using, [10],

$$\sin(x) = x - \frac{x^3}{6}, \quad \cos(x) = 1 - \frac{x^2}{2}. \quad (6)$$

In this case equations (2) and (3) change to

$$P_i = UI_i(1 - \frac{\varphi_p^2}{2}), \quad (7)$$

$$P_{i\varphi} = UI_i \cos(\varphi)(1 - \frac{\varphi_p^2}{2}) - UI_i \sin(\varphi)(\varphi_p - \frac{\varphi_p^3}{6}). \quad (8)$$

Extraction of  $I_i$  from equation (7) yields

$$I_i = \frac{P_i}{U(1 - \frac{\varphi_p^2}{2})} \quad (9)$$

and substitution of equation (9) into (8) leads to the cube equation

$$P_i \sin(\varphi)\varphi_p^3 - 3[P_i \cos(\varphi) - P_{i\varphi}]\varphi_p^2 - 6P_i \sin(\varphi)\varphi_p + 6[P_i \cos(\varphi) - P_{i\varphi}] = 0. \quad (10)$$

The number of real roots of this equation depends on the sign of the expression

$$D = -2 - 6 \left\{ 1 + \frac{[P_i \cos(\varphi) - P_{i\varphi}]^2}{P_i^2 \sin^2(\varphi)} \right\}^2. \quad (11)$$

For  $D < 0$  equation (10) has three real roots.

To solve equation (10) three new expressions have been introduced

$$q = -2 \frac{[P_i \cos(\varphi) - P_{i\varphi}]^3}{P_i^3 \sin^3(\varphi)}, \quad (12)$$

$$r = \pm \sqrt{2 + \frac{[P_i \cos(\varphi) - P_{i\varphi}]^2}{P_i^2 \sin^2(\varphi)}}, \quad (13)$$

$$\cos(\alpha) = \frac{q}{r^3}. \quad (14)$$

The sign of  $r$  must be the same as the sign of  $q$ . If the phase angle  $\varphi_p$  is positive, then  $q$  is negative and vice versa. Three roots of equation (10) are now given by the expressions

$$\varphi_{p1} = -2r \cos\left(\frac{\alpha}{3}\right) + \frac{P_i \cos(\varphi) - P_{i\varphi}}{P_i \sin(\varphi)}, \quad (15)$$

$$\varphi_{p2} = 2r \cos\left(60 - \frac{\alpha}{3}\right) + \frac{P_i \cos(\varphi) - P_{i\varphi}}{P_i \sin(\varphi)}, \quad (16)$$

$$\varphi_{p3} = 2r \cos\left(60 + \frac{\alpha}{3}\right) + \frac{P_i \cos(\varphi) - P_{i\varphi}}{P_i \sin(\varphi)}. \quad (17)$$

Because the expected value of  $\varphi_p$  is near to zero, the proper root must be selected. After finding the value of  $\varphi_p$  from one of equations (15) to (17) the value of the current is calculated from equation (9).

The solution of equation (10) was carried out also in MATLAB. Fig. 2 shows the calculated error of such solution of the parasitic phase shift  $\varphi_p$  and Fig. 3 the error of the solution of the current  $I$ .

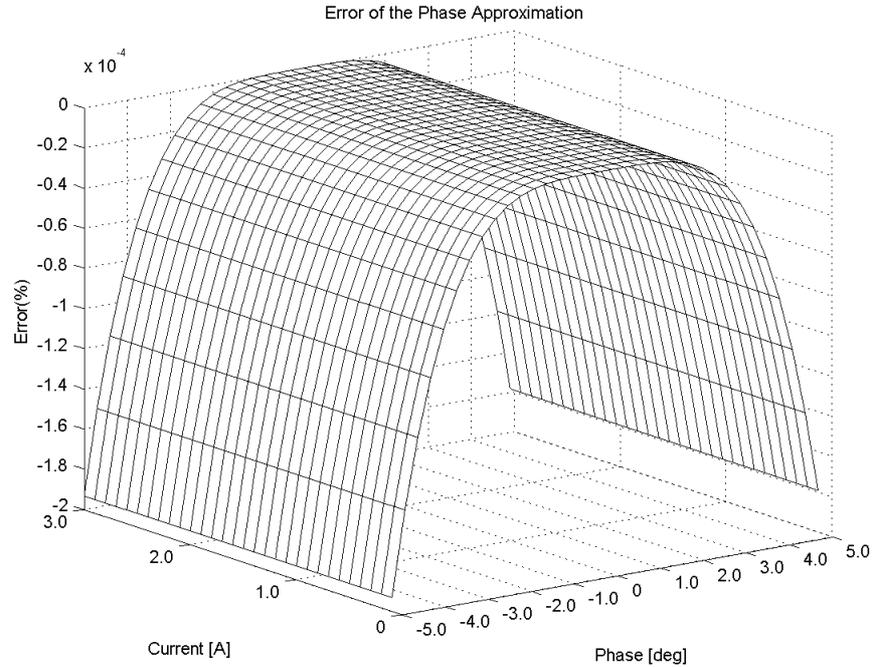


Figure 2. The calculated error of the parasitic phase shift solution

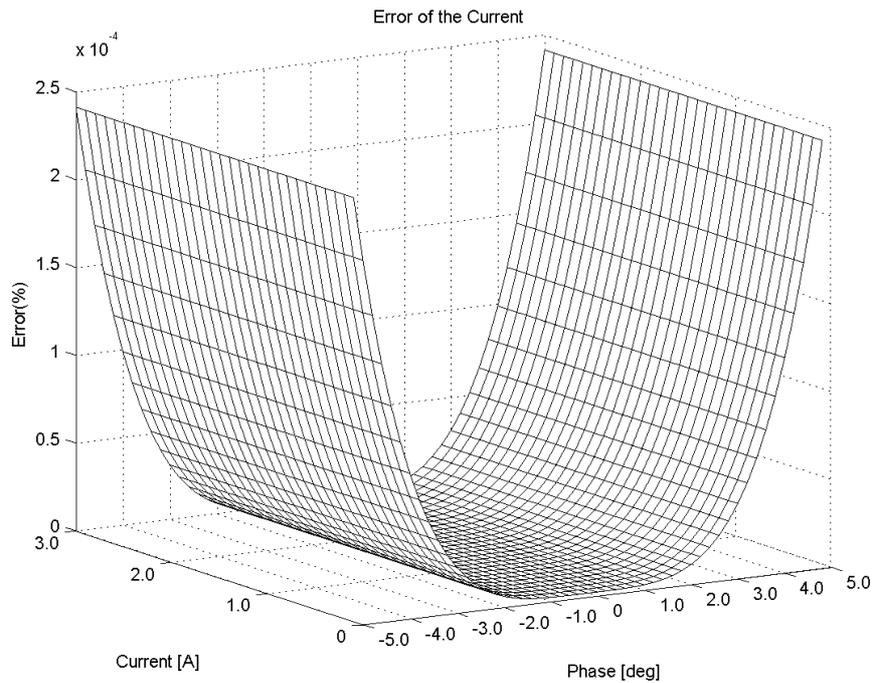


Figure 3. The calculated error of the current solution

It is evident that for  $\varphi_p$  up to  $5^\circ$  the error of such solution is of the order of  $10^{-4}$  per cent. The error is independent on the current value. The results of the solution given by equations (9), (15), (16), (17) are similar.

#### IV. Conclusions

Brief description of the designed digital three-phase electricity meter is given. Modern Texas Instruments TMS320F243 processor and TMS320C6711 DSP were used to get a powerful measuring system. A calibration procedure for the corrections of the measured current and parasitic phase shift between the measured voltage and current based on the active power measurement is given. This procedure enables to calculate the correct values of the current and the phase shift at the same time, thus overcoming the repeating of the current calibration after the phase shift correction. MATLAB calculations were used to simplify current and parasitic phase shift corrections. The errors of such calculations were of the order of  $10^{-4}$  per cent.

This work is supported by the Slovak grant agency GAT under the grant No. 102/VTP/2000.

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