Sensors of impulsive force and pressure with one point and two point strain measurement applied in tasks of reconstruction

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In the paper the structure and the operating principle of impulsive force and pressure sensors applied in tasks of reconstruction is presented, which feature one point and two point measurement of the strain inside a Hopkinson bar type mechanical transducer. A comparison between the two versions of the sensor was carried out, taking into consideration the processing accuracy, the complexity of the conditioning circuit and the input signal processing circuit. The preferred application range is given for each version of the sensor.

Introduction

The Hopkinson bar, with strain gauges mounted upon it (figure 1), is a commonly used sensor in measurements of impulsive force and pressure. The application of the Hopkinson bar as a mechanical transducer in the operating range with dispersion, for measurements of impulsive force and pressure, requires the correcting of dispersive distortion arising during processing. Two groups of methods exist for correcting such distortion: one group, which uses the bar's dispersive characteristic [1], determined analytically or experimentally, and the phase frequency characteristic, determined experimentally [2], and the other group of methods, in which the spectral transmittance of a specified segment of the bar is used, which is determined experimentally [3].

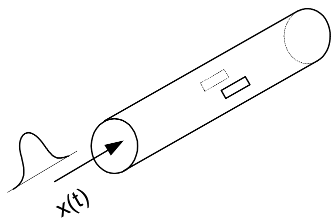

Fig .1. A sketch of the structure of the impulsive force and pressure sensor.

For experimentally determining the bar's dispersive characteristic or the spectral transmittance of a specified segment of the bar the one point or the two point strain measurement is applied [3],[4]. For this purpose mechano-electric transducers are installed in specified places on the bar, thus creating sensors of impulsive force and pressure with either one point or two point strain measurement, devoted to reconstruction of those quantities. In the paper the essence of functioning of the sensors is presented together with examples of force and pressure waveforms reconstructed with their help.

Sensors of impulsive force and pressure with one point and two point strain measurement

The essence of operation of the sensor with one point- and two point strain measurement is best described using the one dimensional equation of the bar's movement and the Lagrangian diagram illustrating the propagation of elastic strain waves inside the bar. The equation of movement of the bar has the form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

where: u is the displacement, $c_0 = (E/\rho)^{0.5}$ is the propagation velocity of elastic strain waves inside the bar, E - Young's modulus and ρ - density of the bar's material.
The solution of equation (1), with the use of discontinuous functions, consists of two functions, which represent waveforms propagating in the positive and the negative direction of the x axes, and which are noted as follows:

$$u(x,t) = u_1\left(t - \frac{x}{c_0}\right) + u_2\left(t + \frac{x}{c_0}\right) \tag{2}$$

The longitudinal strain waves can be written in the form:

$$\varepsilon(x,t) = \varepsilon_1\left(t - \frac{x}{c_0}\right) + \varepsilon_2\left(t + \frac{x}{c_0}\right) \tag{3}$$

where: $\varepsilon(x,t) = \partial u(x,t)/\partial x$
Other mechanical quantities such as mass velocity v, stress σ, can be determined from relations (2), (3).

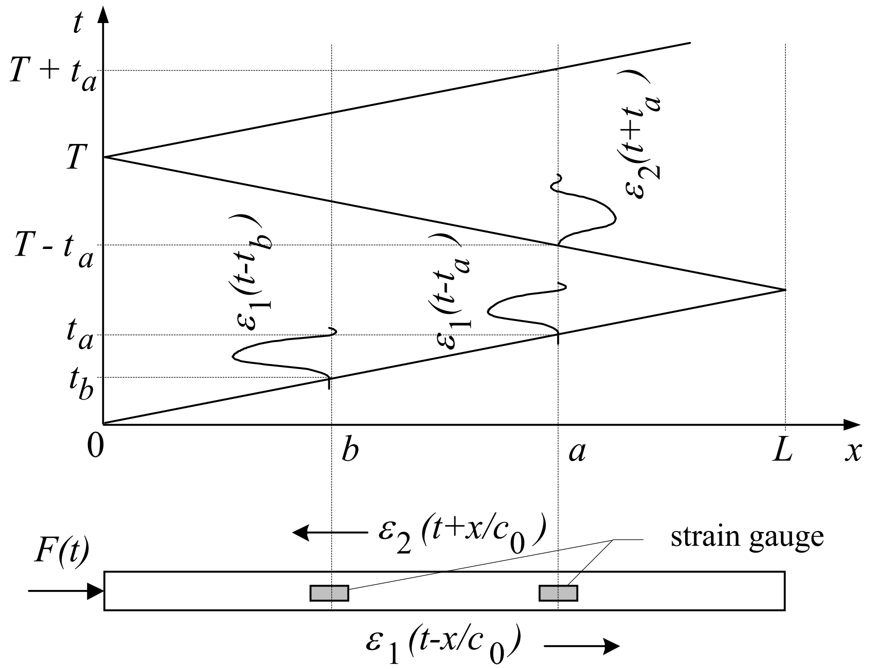

Fig. 2. Lagrangian diagram illustrating the propagation of elastic strain waves inside a bar of length L.

In figure 2 the Lagrangian diagram is presented illustrating the propagation of elastic strain waves inside a bar of length L. The strain gauges are mounted at distances a and b from the left end of the bar, upon which the measured quantity is acting. The wave propagating to the right is denoted as ε_1 while the one propagating to the left as ε_2. Substituting $x = a$ and $x = b$ in equation (3) one obtains:

$$\varepsilon_A(t) = \varepsilon_1(t - t_a) + \varepsilon_2(t + t_a) \tag{4}$$

$$\varepsilon_B(t) = \varepsilon_1(t - t_b) + \varepsilon_2(t + t_b) \tag{5}$$

where: $\varepsilon_A(t) = \varepsilon(a,t)$, $\varepsilon_B(t) = \varepsilon(b,t)$, $t_a = a/c_0$ and $t_b = b/c_0$.
In figure 2, T denotes the time that is needed for the wave to travel twice through the bar $T = 2L/c_0$.

For $t < T - t_a$ the reflected wave does not propagate through the gauges $\varepsilon_2(t + t_a) = 0$ and $\varepsilon_2(t + t_b) = 0$.

From relations (4) and (5) it follows, that:

$$\varepsilon_A(t) = \varepsilon_1(t - t_a) = \varepsilon_{A1}(t) \tag{6}$$

$$\varepsilon_B(t) = \varepsilon_1(t - t_b) = \varepsilon_{B1}(t) \tag{7}$$

In the sensor with two-point strain measurement the waveforms of the incident strain wave are utilized $\varepsilon_{A1}(t)$ and $\varepsilon_{B1}(t)$ measured at points a and b of the bar. For the correct processing of these waveforms by the gauges, two conditions must be satisfied, regarding the strain gauge mounting points, the first one $b \geq 20d$, where d is the diameter of the bar, and the second one $2(L-a)/c_0 > \alpha$, where α- duration of the input quantity. Meeting the first condition ensures the uniformity of the stress distribution in the cross section of the bar, where the gauges are installed, while meeting the second of the conditions prevents the superposition of the incident and the first reflected pulse in the strain gauges. The application of the sensor in reconstruction problems requires satisfying an additional condition: $(a - b) = b$.

For $T - t_a < t < T + t_a$ with $\alpha < T - 2t_a$ the incident wave does not propagate through the strain gauges, $\varepsilon_1(t - t_a) = 0$ and $\varepsilon_1(t - t_b) = 0$. From relations (4) and (5) it results, that:

$$\varepsilon_A(t) = \varepsilon_2(t + t_a) = \varepsilon_{A2}(t) \tag{8}$$

$$\varepsilon_B(t) = \varepsilon_2(t + t_b) = \varepsilon_{B2}(t) \tag{9}$$

For $t > T + t_a$, waveforms of the incident pulse $\varepsilon_1(t - t_a)$ and $\varepsilon_1(t - t_b)$, and waveforms of the reflected one $\varepsilon_2(t + t_a)$ and $\varepsilon_2(t + t_b)$, are nonzero. The waveforms from within this time range are not used in the reconstruction and the dispersive distortion correction processes. In the sensor with one point strain processing waveforms are used which correspond to non interfering waves: the incident $\varepsilon_{A1}(t)$ and the first reflected one $\varepsilon_{A2}(t)$. Restrictions concerning the gauge installation points and the pulse duration for this sensor are as follows: $a \geq 20d$ and $2(L-a)/c_0 > \alpha$. Application of the one point sensor in tasks of reconstruction requires meeting the condition $2(L-a) = a$.

If the Fourier transforms of the incident pulse waveforms $\varepsilon_{A1}(t)$ and $\varepsilon_{B1}(t)$ are designated respectively by $\hat{\varepsilon}_{A1}(j\omega)$ and $\hat{\varepsilon}_{B1}(j\omega)$, than the spectral transmittance of the segment of the bar $(a-b)$ is:

$$G_{(a-b)}(j\omega) = \frac{\hat{\varepsilon}_{A1}(j\omega)}{\hat{\varepsilon}_{B1}(j\omega)} \tag{10}$$

Assuming the homogeneity of the bar's material and the condition, that $(a-b) = b$, it is possible to obtain the Fourier transform of the strain waveform $\hat{\varepsilon}_0(j\omega)$ for $x = 0$ from equation

$$\hat{\varepsilon}_0(j\omega) = \frac{\hat{\varepsilon}_{B1}(j\omega)}{G_{(a-b)}(j\omega)} \tag{11}$$

The waveform $\varepsilon(0,t)$ is obtained using the inverse Fourier transform from formula:

$$\varepsilon(0,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\hat{\varepsilon}_{B1}(j\omega)}{G_{(a-b)}(j\omega)} e^{j\omega t} d\omega \tag{12}$$

The exact result of deconvolution, which arises from the simple division (11) is not practically obtainable because of the existence of noise in the measuring circuit. The best estimate of the convolution component is sought, using regularizational filtering. The strain waveform spectrum estimate is calculated from formula

$$\tilde{\hat{\varepsilon}}_0(j\omega) = \frac{\hat{\varepsilon}_{B1}(j\omega)}{G_{(a-b)}(j\omega)} K(j\omega,\gamma) \tag{13}$$

where $K(j\omega,\gamma)$ is the filter spectral function, and γ - the regularization parameter.
The strain waveform estimate $\tilde{\varepsilon}(0,t)$ is obtained from relation

$$\tilde{\varepsilon}(0,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\hat{\varepsilon}_{B1}(j\omega)}{G_{(a-b)}(j\omega)} K(j\omega,\gamma) e^{j\omega t} d\omega \tag{14}$$

From formulas (10), (13), (14) follows the means of using the sensor with two point strain measurement in tasks of impulsive force and pressure waveform reconstruction.
If the Fourier transforms of the first reflected pulse waveform $\varepsilon_{A2}(t)$ is designated by $\hat{\varepsilon}_{A2}(j\omega)$, than the spectral transmittance of the segment of the bar $2(L-a)$ is:

$$G_{2(L-a)}(j\omega) = -\frac{\hat{\varepsilon}_{A2}(j\omega)}{\hat{\varepsilon}_{A1}(j\omega)} \tag{15}$$

Assuming the homogeneity of the bar's material and the condition, that $2(L-a) = a$, it is possible to reconstruct the estimate of the strain waveform spectrum (Fourier transform) $\tilde{\hat{\varepsilon}}_0(j\omega)$ and the estimate of the strain waveform $\tilde{\varepsilon}(0,t)$ from equations (13) and (14) respectively, by replacing in those equations $\hat{\varepsilon}_{B1}(j\omega)$ with $\hat{\varepsilon}_{A1}(j\omega)$ and $G_{(a-b)}(j\omega)$ with $G_{2(L-a)}(j\omega)$. From such modified equations (13) and (14), and from equation (15) results the way of applying the sensor with one point strain measurement in problems of impulsive force and pressure waveform reconstruction.
The pressure waveform estimate $\tilde{p}(t)$ of the pressure acting on the front face of the bar is obtained from relation $\tilde{p}(t) = \tilde{\varepsilon}(0,t)\cdot E$. The force waveform estimate $\tilde{F}(t)$ is calculated from formula $\tilde{F}(t) = \tilde{\varepsilon}(0,t)\cdot E \cdot A$, where A is the cross section of the bar.

Examples of impulsive force and pressure waveform reconstruction

Examples of reconstruction, with the help of the transformational method, of selected impulsive force and pressure waveforms, making use of the above discussed sensor, are presented in figures 3 – 6. In figures 3 and 4 consecutive stages of reconstruction are displayed, of impact force waveforms, created by a longitudinal impact of a steel sphere into a steel bar, using the one point strain measurement sensor. The technical data concerning the entire measurement circuit and the data acquisition circuit is found in paper [4]. The reconstruction results were verified with respect to duration and maximum value with the Hertz impact theory. A satisfactory consistency was obtained between the reconstruction results and the Hertz theory for spheres of diameters not smaller than $3mm$. For the presented example of impact of a sphere $4.762mm$ in diameter the results are as follows: duration of the incident pulse $\alpha_i = 22\mu s$, duration of the reconstructed pulse $\alpha_r = 13.7\mu s$, duration calculated from the Hertz theory $\alpha_H = 14\mu s$, relative error $(\alpha_r - \alpha_H)/\alpha_H = 2.1\%$ and the maximum value of the force results respectively $F_{mr} = 265N$, $F_{mH} = 258$, relative error $(F_{mr} - F_{mH})/F_{mH} = 3\%$.

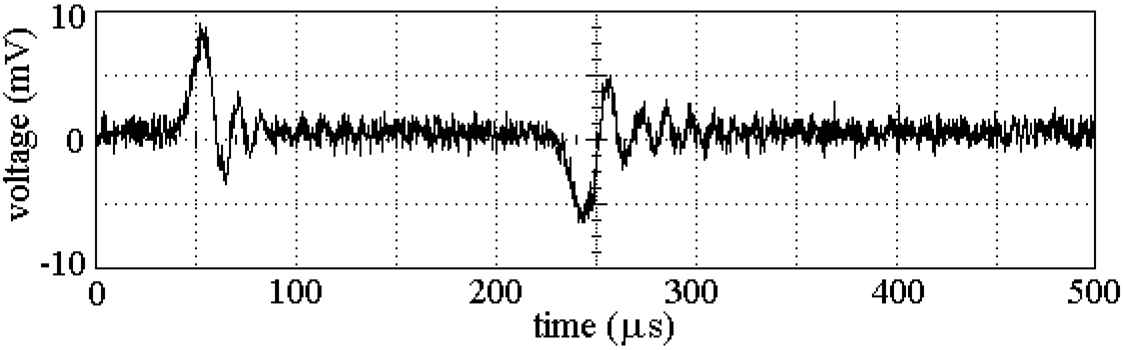

Fig. 3. The output voltage waveform of the sensor with one point strain measurement, created as a result of a longitudinal impact of a steel sphere into a steel bar. The length of the bar $L = 1{,}5m$, the bar's diameter $22mm$, sphere diameter $4.762mm$, distance $a = 100cm$, impact velocity $V = 2.2m/s$.

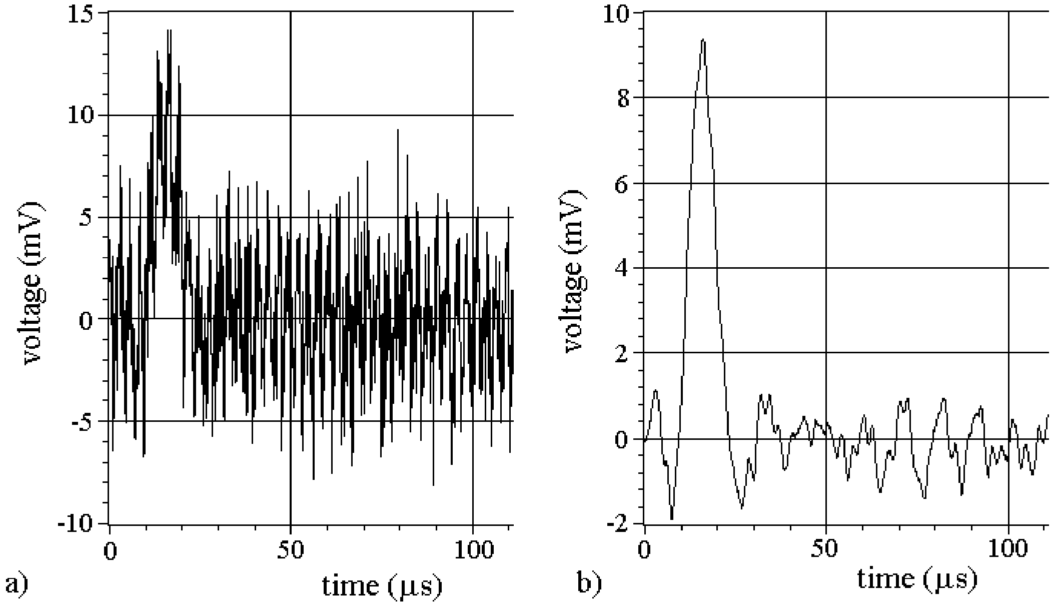

Fig. 4. The illustration of the stages of reconstructing impact force waveforms, c) input pulse reconstructed without regularization, d) the input quantity waveform reconstructed with regularization; value of the regularization parameter $\gamma = 0.003$.

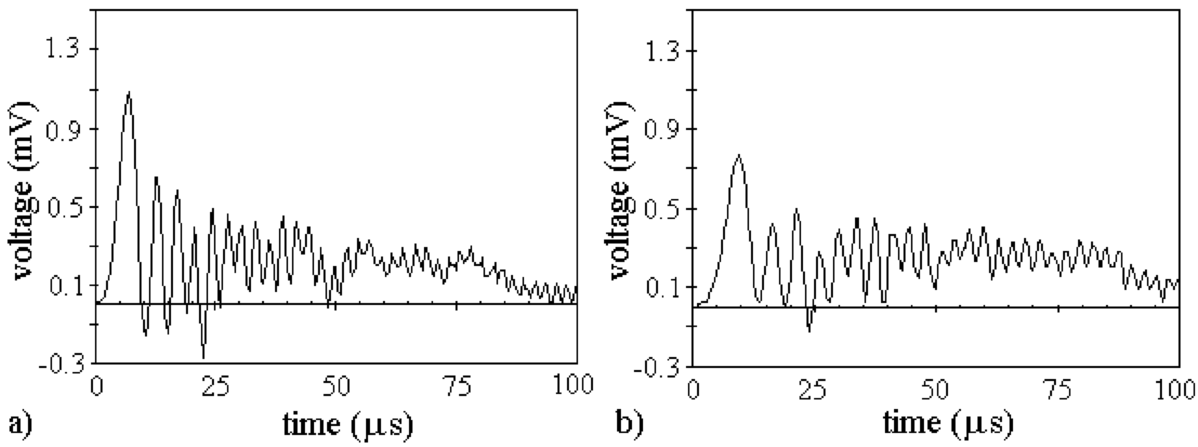

Fig. 5. The output signal of the strain gauge transducer situated: a) at distance $b = 1m$, b) at distance $b = 2m$. The sensor with two point strain measurement.

In figures 5 and 6 the reconstruction process of impact pressure waveform, created by an electrical discharge in water, is pictured, making use of the sensor with two point strain measurement. The technical data concerning the entire measurement circuit and the data acquisition circuit is included in paper [3]. The obtained reconstruction results are consistent to a satisfactory extent with regard to duration and amplitude with results obtained by other authors using a different method.

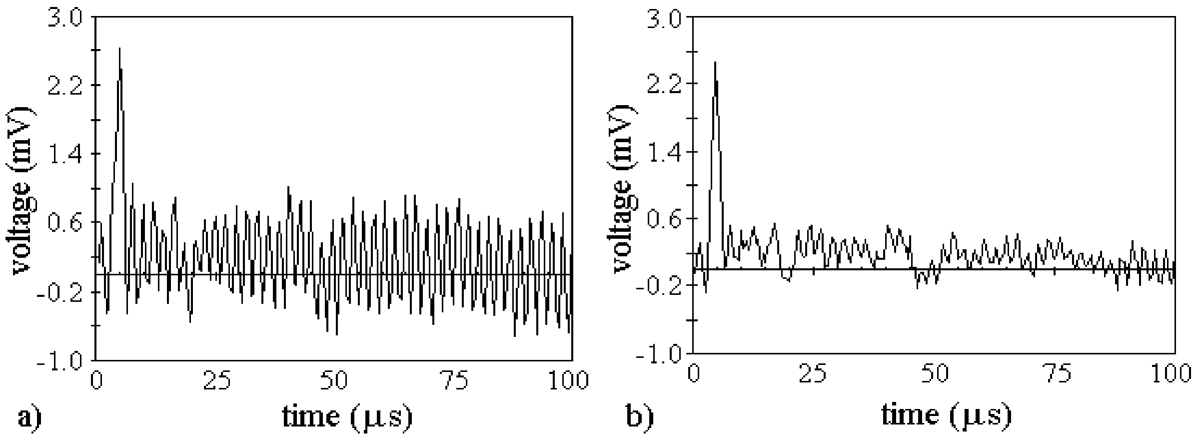

Fig. 6. The reconstructed output signal from a hypothetical strain gauge transducer situated at the beginning of the bar: c) before the regularizational filtering process, d) after the regularizational filtering process (value of the regularization parameter $\gamma = 0.00072$).

Summary

By introducing limitations concerning the strain gauges localization and restrictions concerning the duration of the input quantity, it is possible to construct impulsive force and pressure sensors with one point and two point strain measurement, intended for reconstruction problems. The sensors with one point strain measurement are used for reconstruction of impulsive quantities of strictly defined duration, e.g. for reconstruction of impulsive force waveforms generated by mechanical impact. The sensors with two point strain measurement are applied for input quantities, which duration is difficult to strictly specify, or which occur together with other waveforms generated by the object. An example of application of such a sensor is the reconstruction of impact pressure waveforms created by electrical discharge in water. The advantages of the sensor with one point processing are its cooperation with a single channel amplification and signal conditioning circuit and the greater ease of achieving the required processing accuracy and therefore the required reconstruction accuracy. This last quality is particularly crucial when using semiconductor strain gauges featuring significant spread of the sensitivity coefficient and resistance.

References

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