

A PROPOSAL OF AN EVALUATION METHOD OF UNCERTAINTY IN THE TRACEABILITY SYSTEM OF FORCE

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Abstract:

This paper proposes a new approach for evaluating the uncertainty in the traceability system of force with the following features. a) Repeatability/reproducibility is evaluated by the Type A evaluation. b) Data taken in each orientation is treated with equality. c) Deviations from reference values and from the interpolation equation are considered to be equal to the standard deviation 1σ of a normal distribution. d) Difference of the values in increasing load and in decreasing load are considered as a half-width of the possible distribution of values to be measured. The same is the case of difference of the zero points in a loading cycle.

1. Introduction

A leading and laborious work[1],[2] has been done for establishing the evaluation methods of uncertainty in the calibration of force calibration machines and force-measuring devices. However, the authors faced a sort of difficulty in some actual cases, that is, there was a concern that the uncertainty might be underestimated in some cases when applying the above mentioned evaluation method. Thus, the authors came to reconsider the method. This paper proposes a new method for evaluating uncertainty in the traceability system of force. The method tries to accord with the way of uncertainty evaluation stated in the GUM[3].

It should be noted that all of the uncertainty factors discussed below are relative values.

2. Uncertainty in the comparison calibration of a force calibration machine by force standard machines

Force calibration machines (FCMs) are “calibrated” by making comparisons with force standard machines (FSMs), the national standard. The comparisons are carried out within a short period of time using a series of high-precision force transducers as transfer standards. In this chapter, uncertainty of the comparison, and accordingly the uncertainty of the FCMs, are evaluated.

2.1 Uncertainty of the reference values determined by the FSM

Uncertainty of the reference values obtained from the calibrations of the transfer standards by the FSMs, arises from both uncertainty due to the FSMs and due to the transfer standards.

2.1.1 Combined standard uncertainty of the FSM u_{c_fsm}

Uncertainty in the realization of the unit of force is evaluated for each FSM by considering various uncertainty factors such as mass of deadweights, gravitational acceleration at the location of the FSM, and amplification factor (in case of FSM of force amplification types). It is confirmed by comparisons with other FSMs or by inter-comparisons with FSMs of national metrology institutes (NMIs) in other countries. The followings are typical values of uncertainty of FSMs.

$$u_{c_fsm} = 0.0010 \% \quad (\text{for FSM of the deadweight type}) \quad (2.1)$$

$$u_{c_fsm} = 0.0025 \% \text{ to } 0.0050\% \quad (\text{for FSM of the hydraulic or lever amplification type}) \quad (2.2)$$

2.1.2 Uncertainty originated in the transfer standard

Uncertainty factors comes from the transfer standard are dispersion of measured values, resolution of the indicator and sensitivity drift of the transfer standards during the period of comparison. Other factors such as zero point shift and hysteresis are discussed in section 2.2.

2.1.2-a Standard uncertainty from repeatability / reproduceability at the FSM $u_{_tsd_rep}$

At each force step, influence of dispersion of values X_{ij} measured by the FSM can be evaluated as Type A uncertainty with the following equation, where X_{ij} is the measured value of the i -th loading cycle out of m times loadings in the j -th orientation out of n orientations of the transfer standard, and m_x is an average of X_{ij} from $m \times n$ times measurements. It should be noted that data taken in each orientation is treated with equality.

$$u_{_tsd_rep} = \sqrt{\frac{1}{mn(mn-1)} \sum_{i=1}^m \sum_{j=1}^n \left(\frac{X_{ij} - m_x}{m_x} \right)^2} \quad (2.3)$$

2.1.2-b Standard uncertainty due to the resolution of the transfer standard $u_{_tsd_res}$

Standard uncertainty of resolution of the indicator of the transfer standard $u_{_tsd_res}$ is evaluated by assuming a rectangular distribution having width of f_{r_tsd} . The relative resolution f_{r_tsd} is one

increment of the smallest digit of the indicator divided by an average of the indicated values at each force step. If the indication fluctuates under null loading, the one increment shall be replaced by a peak-to-peak value of the fluctuation.

$$u_{_tsd_res} = (1/\sqrt{3})(f_{r_tsd} / 2) \quad (2.4)$$

2.1.2-c Standard uncertainty due to the sensitivity drift of the transfer standard $u_{_tsd_drf}$

Transfer standards are calibrated by the FSMs before and after the on-site measurement at the FCM, and reference value at each force step is determined by taking an average of values of the pre-calibration and the post-calibration. Influence of the sensitivity drift of the transfer standard can be evaluated by assuming a rectangular distribution having width of $b_{_tsd_drf}$, where relative drift $b_{_tsd_drf}$ is the sensitivity difference between the pre-calibration and the post-calibration divided by an average of the values at each force step.

$$u_{_tsd_drf} = (1/\sqrt{3})|b_{_tsd_drf} / 2| \quad (2.5)$$

Finally combined standard uncertainty originated in the transfer standard u_{c_tsd} is given as follows with taking above mentioned factors into account.

$$u_{c_tsd} = \sqrt{u_{_tsd_rep}^2 + u_{_tsd_res}^2 + u_{_tsd_drf}^2} \quad (2.6)$$

2.1.3 Uncertainty of the reference value determined by the FSM

Combined standard uncertainty of the reference value determined by the FSM u_{c_ref} is then calculated from the uncertainty of the FSM u_{c_fsm} and the uncertainty originated in the transfer standard u_{c_tsd} which are discussed in the previous sections.

$$u_{c_ref} = \sqrt{u_{c_fsm}^2 + u_{c_tsd}^2} \quad (2.7)$$

2.2 Uncertainty of the measured values by the FCM

Followings should be considered in the measurements of the transfer standards by the FCM.

2.2.1-a Standard uncertainty from repeatability / reproduceability at the FCM $u_{_fcm_rep}$

Influence of dispersion of the measured values X'_{ij} by the FCM can be evaluated as Type A uncertainty in the same manner as stated in section 2.1.2-a.

$$u_{_fcm_rep} = \sqrt{\frac{1}{mn(mn-1)} \sum_{i=1}^m \sum_{j=1}^n \left(\frac{X'_{ij} - m_{x'}}{m_{x'}} \right)^2} \quad (2.8)$$

2.2.1-b Standard uncertainty due to the resolution of the transfer standard $u_{_fcm_res}$

Standard uncertainty from resolution of the transfer standard $u_{_fcm_res}$ is again taken into account in the measurement by the FCM in the same manner as stated in section 2.1.2-b.

$$u_{_fcm_res} = (1/\sqrt{3})(f_{r_tsd} / 2) \quad (2.9)$$

2.2.1-c Standard uncertainty caused from difference of zero point shifts of the reference standards observed by the FSM and by the FCM $u_{_fcm_zer}$

In usual, zero point of the indicator after a loading cycle shifts from that before loading. This is characteristic of the transfer standard. However, if we compare the zero point shifts observed by the FCM with that by the FSM, then the characteristic of the transfer standard is cancelled out and hysteresis characteristic of the FCM is revealed as a difference between the zero point shifts observed by the FSM and by the FCM. Here it is assumed that the initial zero point is located with probability of a normal distribution shown in Fig.1. This normal distribution is assumed to be centered on the zero point before loading, and to have a width of $3s$ which corresponds to value of the zero point shift. Standard uncertainty caused from the difference of zero point shifts is evaluated as follows, where b_{0_fcm} and b_{0_fsm} are the zero point shifts relative to the maximum capacity of the measuring range at the FCM and at the FSM, respectively.

$$u_{_fcm_zer} = |(b_{0_fcm} / 3) - (b_{0_fsm} / 3)| \quad (2.10)$$

2.2.1-d Standard uncertainty caused from the difference of hysteresis of the reference standards observed by the FSM and by the FCM $u_{_fcm_hys}$

Indication of the transfer standard at a force step in increasing load is generally not coincide with that at the same force step in decreasing load, in other words, the transfer standard exhibits a hysteresis characteristic. However, it is thought that difference of the hysteresis observed by the FSM and that by the FCM shows a characteristic of the FCM because the characteristic of the transfer standard itself is canceled out by taking the difference. Here it is also assumed that

possible distribution of the value to be measured at a force step in increasing load is represented by a normal distribution shown in Fig.1. This normal distribution is assumed to be centered on the indicated value in increasing load, and to have a width of $3s$ which corresponds to the difference between the indicated value at the force step in increasing load and that in decreasing load. The same is the case for the value to be measured at a force step in decreasing load. Standard uncertainty caused from the difference of hysteresis is evaluated as follows, where n_{fcm} and n_{fsm} are the relative hysteresis observed by the FCM and by the FSM, respectively. The relative hysteresis n is a difference between the indicated values in increasing load and in decreasing load divided by an average of the indicated values in increasing load.

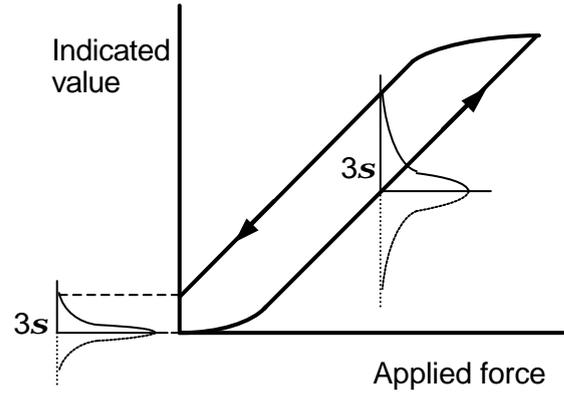


Figure 1 Zero point shift and hysteresis of a force transducer (transfer standard)

$$u_{fcm_hys} = \left| \left(n_{fcm} / 3 \right) - \left(n_{fsm} / 3 \right) \right| \quad (2.11)$$

2.2.1-e Standard uncertainty due to the deviation from the reference value u_{fcm_dev}

At each force step, the measured value by the FCM usually shows some deviation from the reference value given by the FSM. Influence of the deviation is evaluated as follows by assuming that the deviation is equivalent to standard deviation $1s$ of a normal distribution, where b_{d_fcm} is the deviation divided by an average of the indicated values in increasing load.

$$u_{fcm_dev} = |b_{d_fcm}| \quad (2.12)$$

Finally combined standard uncertainty of the measured value by the FCM u_{c_fcm} is given as follows with taking above mentioned factors into account.

$$u_{c_fcm} = \sqrt{u_{fcm_rep}^2 + u_{fcm_res}^2 + u_{fcm_zer}^2 + u_{fcm_hys}^2 + u_{fcm_dev}^2} \quad (2.13)$$

As for the comparator type FCM, other factors due to the use of a reference force transducer should be additionally taken into account. They are uncertainty in the calibration results of the reference transducer, uncertainty due to temperature fluctuation during the use of the reference transducer, and uncertainty due to long-term instability of the reference transducer.

2.3 Uncertainty of the comparison of the FCM with the FSM

Expanded uncertainty of the comparison of the FCM with the FSM U_{cmp} is calculated from the uncertainty of the reference value by the FSM u_{c_ref} and the uncertainty of the measured value by the FCM u_{c_fcm} . In usual coverage factor $k=2$ is used for the confidence level of 95%.

$$U_{cmp} = k \sqrt{u_{c_ref}^2 + u_{c_fcm}^2} \quad (2.14)$$

2.4 Uncertainty of the FCM confirmed through the comparison with the FSM

With the current technology, capability of a FCM is confirmed only through comparisons with FSMs, although the FCM may have better measurement capability than that confirmed by the comparison. Expanded uncertainty of the FCM U_{cal_fcm} with coverage factor $k=2$ is estimated to be equal to or smaller than the expanded uncertainty of the comparison U_{cmp} .

$$U_{cal_fcm} \leq U_{cmp} \quad (2.15)$$

3. Uncertainty in the calibration of a force transducer by a force calibration machine

Calibrations of force transducers are generally carried out following the procedure described in ISO 376[4]. Force transducers are tested and classified according to ISO 376 as well. However, the current version of ISO 376 does not describe how to evaluate the uncertainty of the calibration results. In this chapter, a method for uncertainty evaluation of the calibration results of force transducers is described adhering to the rules of uncertainty expression stated in the GUM[3].

3.1 Uncertainty of the FCM

Uncertainty in the realization of the unit of force by a FCM is evaluated through comparisons with FSMs. Combined standard uncertainty of the FCM $u_{c_cal_fcm}$ is given as follows.

$$u_{c_cal_fcm} = U_{cal_fcm} / 2 \quad (3.1)$$

3.2 Uncertainty originated in the force transducer

Following factors should be considered as the uncertainty coming from the force transducer under calibration. If the force transducer has an interpolation equation, residues in the interpolation should be added to these factors.

3.2-a Standard uncertainty from reproduceability (with orientation change) u_{tra_rep}

At each force step, influence of dispersion of the measured values X_i can be evaluated as Type A uncertainty, where X_i is the value measured in the first loading cycle in the i -th orientation out of n orientations of the force transducer, and m_x is an average of X_i of n times measurements.

$$u_{tra_rep} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n \left(\frac{X_i - m_x}{m_x} \right)^2} \quad (3.2)$$

Note that data taken in the second cycle in the orientation of 0 degree is not used so that data taken in each orientation is treated with equality. It is considered that influence of dispersion of the measured values is well represented by the reproducibility of data taken from different orientations. Thus the repeatability of data measured in the first orientation is not taken into account to avoid an overestimation of the uncertainty.

3.2-b Standard uncertainty due to the resolution u_{tra_res}

Standard uncertainty due to the resolution of the indicator of the force transducer u_{tra_res} is evaluated in the same manner as stated in section 2.1.2-b, provided that only the data of the first loading cycles in each orientation is used for directional equality.

$$u_{tra_res} = (1/\sqrt{3})(f_{r_tra} / 2) \quad (3.3)$$

3.2-c Standard uncertainty due to the zero point shift u_{tra_zer}

Standard uncertainty due to the zero point shift of the force transducer u_{tra_zer} is evaluated under the same consideration as stated in section 2.2.1-c (also see Fig.1), where b_{0_tra} is an average of zero point shifts of the first loading cycles in each orientation divided by the maximum capacity of the force transducer.

$$u_{tra_zer} = |b_{0_tra}| / 3 \quad (3.4)$$

3.2-d Standard uncertainty due to the hysteresis u_{tra_hys}

As hysteresis is thought to reflect a time-dependent characteristic of the force transducer, its influence on the calibration results should be considered even if the force transducer is used for measurements in increasing load only. Standard uncertainty due to the hysteresis u_{tra_hys} is evaluated under the same consideration as stated in section 2.2.1-d (also see Fig.1), where n_{tra} is an average of relative hysteresis in each orientation. Note that the relative hysteresis is relative to the force level at each force step.

$$u_{tra_hys} = |n_{tra}| / 3 \quad (3.5)$$

3.2-e Standard uncertainty due to the interpolation u_{tra_int}

For the force transducers having an interpolation equation, another uncertainty factor should be considered. The interpolation equation is in general determined by applying the least-square-sum method. Residue in the interpolation is considered to correspond to the standard deviation $1s$ of a normal distribution. Influence of the interpolation is evaluated as follows, where b_{d_tra} is the relative deviation of the calculated value by the interpolation equation from an average of the measured values of the first loading cycles in each orientation. Note that the relative deviation is relative to the force level at each force step.

$$u_{tra_int} = |b_{d_tra}| \quad (3.6)$$

Finally combined standard uncertainty originated in the force transducer u_{c_tra} is given as follows with taking above mentioned factors into account.

For a force transducer with an interpolation equation:

$$u_{c_tra} = \sqrt{u_{tra_rep}^2 + u_{tra_res}^2 + u_{tra_zer}^2 + u_{tra_hys}^2 + u_{tra_int}^2} \quad (3.7)$$

For a force transducer without an interpolation equation:

$$u_{c_tra} = \sqrt{u_{tra_rep}^2 + u_{tra_res}^2 + u_{tra_zer}^2 + u_{tra_hys}^2} \quad (3.7')$$

3.3 Uncertainty of results of the calibration of the force transducer

Expanded uncertainty of the calibration results of the force transducer U_{cal_tra} is calculated from the

uncertainty of the FCM $u_{c_cal_fcm}$ and the uncertainty due to the force transducer under calibration u_{c_tra} . In usual coverage factor $k=2$ is used for the confidence level of 95%.

$$U_{_cal_tra} = k\sqrt{u_{c_cal_fcm}^2 + u_{c_tra}^2} \quad (3.8)$$

4. Uncertainty in the calibration of a force-measuring system of an uniaxial testing machine by a force transducer

Calibration of force-measuring systems of uniaxial testing machines(UTMs), is carried out with the procedure described in ISO 7500-1[5]. Although discussions are going on how to evaluate the uncertainty of the calibration results of UTMs, the current version of ISO 7500-1 does not describe the uncertainty evaluation method. In this chapter, a method for uncertainty evaluation of the calibration results is described in accordance with the GUM[3].

Due to the limitation of space, uncertainty evaluation is not discussed here for the case in which reference weights are used as the transfer standard.

4.1 Uncertainty of the reference value

When evaluating uncertainty of the reference values, the following factors arising in the use of the force transducer should be taken into account in addition to the calibration uncertainty.

4.1-a Combined standard uncertainty of the reference force transducer $u_{c_cal_tra}$

Combined standard uncertainty of the calibration result of the force transducer can be derived from the expanded uncertainty $U_{_cal_tra}$ described in the calibration certificate of the force transducer.

$$u_{c_cal_tra} = U_{_cal_tra} / 2 \quad (4.1)$$

4.1-b Standard uncertainty due to the temperature fluctuation during use $u_{_tra_tmp}$

First of all, compensation shall be done for the temperature difference between at the time of the calibration(18°C to 28°C, ± 1 K [4]) and at the time of use(10°C to 35°C, ± 2 K [5]) of the force transducer. After the compensation, influence of temperature fluctuation $Dt_{_cal_utm}$ during the calibration of the UTM should be considered. Here, it is assumed that the temperature fluctuation is represented by a rectangular probability distribution having width of $Dt_{_cal_utm}$,

$$u_{_tra_tmp} = (1/\sqrt{3})a|Dt_{_cal_utm} / 2| \quad (4.2)$$

where a is the temperature sensitivity coefficient(relative) of the force transducer.

4.1-c Standard uncertainty due to the long-term stability $u_{_tra_stb}$

Uncertainty due to the long-term stability of the force transducer can be evaluated either as Type A or as Type B. If the force transducer was calibrated $n_{cal}(>2)$ times, then this influence is evaluated as the same magnitude as the standard deviation of the calibration results $X_{cal,i}$,

$$u_{_tra_stb} = \sqrt{\frac{1}{n_{cal}(n_{cal}-1)} \sum_{i=1}^{n_{cal}} \left(\frac{X_{cal,i} - m_{x_{cal}}}{m_{x_{cal}}} \right)^2} \quad (4.3)$$

where $m_{x_{cal}}$ is an average of $X_{cal,i}$ of n_{cal} times calibrations. If the force transducer is calibrated only a few times, then the uncertainty is temporarily represented by a typical value of the instability of force transducers (for instance, 2×10^{-4} for one calibration interval).

Finally combined standard uncertainty of the reference value determined by the force transducer u_{c_ref} is given as follows with taking above mentioned factors into account.

$$u_{c_ref} = \sqrt{u_{c_cal_tra}^2 + u_{_tra_tmp}^2 + u_{_tra_stb}^2} \quad (4.4)$$

4.2 Uncertainty originated in the UTM

Following factors should be considered as uncertainty factors coming from the UTM.

4.2-a Standard uncertainty from repeatability / reproduceability $u_{_utm_rep}$

At each force step, influence of dispersion of the measured values X_i can be evaluated as Type A uncertainty, where X_i is the value measured in the i -th loading cycle and m_x is an average of X_i from n times measurements.

$$u_{_utm_rep} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n \left(\frac{X_i - m_x}{m_x} \right)^2} \quad (4.5)$$

4.2-b Standard uncertainty due to the resolution $u_{_utm_res}$

Standard uncertainty due to the resolution of the indicator of the UTM $u_{_utm_res}$ is evaluated from the relative resolution f_{r_utm} of the UTM in the same manner as stated in section 2.1.2-b.

$$u_{_utm_res} = (1/\sqrt{3})f_{r_utm} / 2 \quad (4.6)$$

4.2-c Standard uncertainty due to the zero point shift u_{utm_zer}

Standard uncertainty due to the zero point shift of the UTM u_{utm_zer} is evaluated under the same consideration as stated in section 2.2.1-c (also see Fig.1), where b_{0_utm} is an average of zero point shifts of all cycles divided by the maximum capacity of the range under calibration.

$$u_{utm_zer} = |b_{0_utm}|/3 \quad (4.7)$$

4.2-d Standard uncertainty due to the hysteresis u_{utm_hys}

Since hysteresis is thought to reflect a time-dependent characteristic of the UTM, it is desirable to consider influence of the hysteresis even if the UTM is used for measurements in increasing load only. Standard uncertainty due to the hysteresis of the UTM u_{utm_hys} is evaluated under the same consideration as stated in sections 2.2.1-d and 3.2-d (also see Fig.1), where n_{utm} is an average of relative hysteresis of all loading cycles. Note that the relative hysteresis is relative to the force level at each force step.

$$u_{utm_hys} = |n_{utm}|/3 \quad (4.8)$$

Finally combined standard uncertainty originated in the force transducer u_{c_utm} is given as follows with taking above mentioned factors into account.

$$u_{c_utm} = \sqrt{u_{utm_rep}^2 + u_{utm_res}^2 + u_{utm_zer}^2 + u_{utm_hys}^2} \quad (4.9)$$

4.3 Uncertainty of results of the calibration of the force-measuring system of the UTM

Expanded uncertainty of the calibration result of the UTM U_{cal_utm} is calculated from the uncertainty of the reference value by the force transducer u_{c_ref} and the uncertainty originated in the UTM under calibration u_{c_utm} . In usual coverage factor $k=2$ is used for the confidence level of 95%.

$$U_{cal_utm} = k\sqrt{u_{c_ref}^2 + u_{c_utm}^2} \quad (4.10)$$

Note: Uncertainty in using the calibrated UTM

Although "relative accuracy error q " is treated as one of the characteristics of an UTM in ISO 7500-1[5], it represents relative deviation from the reference value and it should be considered as the calibration result. If this deviation cannot be compensated for when using the calibrated UTM, its influence u_{utm_dev} should be evaluated as follows and should be taken into account.

$$u_{utm_dev} = |q| \quad (4.11)$$

This factor should be reflected on uncertainty $u_{c_use_utm}$ of the measurement results obtained by using the UTM along with other factors, which is associated with the use of the UTM, such as temperature fluctuation during the measurement u_{utm_tmp} , long-term stability of the force-measuring system of the UTM u_{utm_stb} , misalignment of a test piece and fixtures u_{utm_ali} and so on.

$$u_{c_use_utm} = \sqrt{u_{c_cal_utm}^2 + (u_{utm_dev}^2) + u_{utm_tmp}^2 + u_{utm_stb}^2 + u_{utm_ali}^2 + \dots} \quad (4.12)$$

5. Concluding remarks

A new approach for evaluating uncertainty in the traceability system of force is proposed. The authors would be happy if this proposal could contribute to the establishment of the coherent and comprehensive evaluation methods of uncertainty in the traceability system of force.

References

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