

Height Difference Measurement in PTB's Liquid Column Manometer

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Abstract – A primary liquid column manometer (LCM) is under development at PTB for use in conducting low-pressure measurements up to 2 kPa in gauge and absolute pressure. To calculate pressure, all measured input quantities of the instrument, as there are liquid density, gravitational acceleration, and length, are traceable to the International System of Units (SI), thus making the LCM a primary pressure standard. The LCM is well suited to identifying small force-induced errors, particularly those of force-compensated pressure balances with non-rotating piston in their lower measurement range, and as such to disseminating the pascal, the SI unit of pressure. This report focuses on the measurement of the height difference inside the instrument, where homodyne plane mirror interferometry is applied using the liquid's free surface as the reflecting mirror for the laser beam.

I. INTRODUCTION

LCMs realise pressure by sensing the displacement of two columns of a liquid in a U-shaped tube, starting from a zero condition where the pressure in both tubes is equalised. A pressure in one tube can then be derived by the fundamental physical relation $p = \rho_l g h$, where p is the measured pressure difference between both tubes, ρ_l is the density of the liquid, g the gravitational acceleration, and h the height difference between the two liquid levels.

The first key feature of the instrument is the use of vacuum oil as manometric liquid inside the tube. Those oils are characterised by a low vapor pressure – that makes the measurement of small absolute pressures realisable. They are in comparison to mercury, a liquid often used in other LCMs, non-poisonous and one magnitude less dense, and support with this the pressure to length sensitivity of the instrument, a way to overcome the weakness in zero pressure reproducibility that was reported for other LCMs. On the other hand, the thermal expansion coefficients of vacuum oils are higher and they absorb gases, both effects could show significant influence on the liquid density [1], a possible source of drift or instability of the instrument. A

solution that was realised the first time, is the direct measurement of the liquid density *in-situ*, what is the second key feature of the instrument.

The search for an adequate liquid to be used inside the instrument was the beginning of the development process. A stepwise investigation starting with ten candidate liquids was performed as part of the *pres2vac* project¹, with the aim to determine the oil's properties of metrological interest. These are viscosity, compressibility, surface tension, wettability, gas absorption capacity and vapour pressure. As the result a synthetic hydrocarbon known as Edwards 45, was chosen, mainly due to its pureness, respectively low vapour pressure, and less water vapour absorption characteristic [2].

To answer the question how the input quantities could be measured accurately, possible methodologies were analysed. By that the value of g was determined sufficiently by a single onsite measurement and under making use of portable standards. Two concepts for the *in-situ* measurements of the liquid density ρ_l were taken into account, the direct measurement of the liquid density under use of an oscillating u-tube densimeter and the temperature-of-flotation method (TFM), where a temperature is varied to find the flotation state of a well-known solid density standard inside a liquid, whose density is similar to the density of the liquid. When floating, the density of the liquid and the density of the standard is equal [3]. The latter method was chosen as it allows a direct integration into the pressure measurement process and its systematics follow strong fundamental principles.

This report focuses on the measurement of the height difference h involving two homodyne differential interferometers produced by SIOS Meßtechnik of Germany that measure the liquid column displacement and hence the difference in height between the two liquid columns of the LCM.

¹ A project within the EMPIR, jointly funded by the EMPIR participating countries within EURAMET and

the European Union.

II. LENGTH MEASUREMENT

A. Instrument integration

For PTB's LCM, both tubes of the U-tube setup are mounted in a hanging configuration under a massive traverse made of granite. The entire instrument rests on a decoupled foundation to isolate it from vibrations. For the same reason, all connections to the U-tube from the pumps and larger valves needed for the inlet and outlet of gases are realised as flexible corrugated hoses in a hanging configuration from a wall-mounted support.

B. Interferometer

The two viewports form the upper end of the tubes and are mounted together on the lower side of the granite traverse. The interferometers are affixed to the upper side of the traverse but sunk into a central recess where they are positioned just a few millimetres above the viewports. This arrangement realises a short interferometric measured length as well as an indirect mechanical coupling of the two components to reduce vibrational influences from the tubes on the interferometers. Both beams (the measurement beam and the reference beam) from each interferometer can pass through the viewport window into the interior of the tubes. In each tube, the measurement beam is directed at the liquid's surface and the reference beam is reflected by a mirroring surface below the viewport window.

Compared to a single beam configuration, the differential configuration of the interferometers has the advantage of reducing the dead-path in the measured length. Moreover, it compensates for ambient effects in the beam paths between the interferometer and the viewport window, and in the viewport window itself, as in this section of the path both beams pass through the same media.

Each interferometer uses a fibre coupled He-Ne laser light source that delivers laser light with a 633 nm wavelength characteristic. The only interferometer parts mounted next to the U-tube setup are the polarising beam splitter and the signal receiver. This arrangement prevents thermal loads and enables a compact design.

C. Influences on the length measurement

The separation of the interferometers from the tubes combined with the coupling to the granite structure is advantageous with respect to minimising length measurement errors caused by tube bending or tilting. Such errors affect length measurement if the interferometers, and with them their measurement beam or reference beam, change their orientation in the time between the zero condition (equal pressure in both tubes) and the time of pressure measurement.

A slight tilting of the tubes can even be observed when gas is introduced into one tube for the pressure measurement. The reason for this can be found when the material

deformations were analysed. Inside the wall material of both tubes a biaxial stress state can be assumed, where the axial elongation of the tubes is derived from the knowledge of the Young's modulus and the Poisson ratio of the wall material and the axial and tangential stresses. The axial stress is driven by the hydrostatic pressure at the inner bottom part of the connected, liquid-containing tubes and is the same in both tubes. Whereover, due to the height dependence of the pressure, the tangential stress changes along the tube and its distribution differs in each tube during the pressure measurement, except the situation when the pressure in both tubes is equalised.

For the example presented in Fig. 1, where in comparison to the reference tube, higher tangential stresses were applied over a wide range of the pressurised tube, that were causing a small axial contraction in addition to the main effect of circumferential elongation of the tube. That leads into the phenomena, that the pressurised tube gets less elongated compared to the reference tube. As both tubes are connected, the U-tube setup tilts (see Fig. 1b). The difference in tube length Δl can be calculated by Eq. 1, with negative values indicating that the pressurised tube is shorter than the reference tube:

$$\Delta l = -\frac{d \cdot \nu \cdot l}{4 \cdot s \cdot E_s} p = -6.4 \text{ nm}, \quad (1)$$

valid for a stainless-steel tube at 20 °C, pressurised at $p = 2 \text{ kPa}$, with a Young's modulus $E_s = 200 \text{ GPa}$, a Poisson ratio $\nu = 0.277$, a length $l = 460 \text{ mm}$, an inner diameter $d = 60 \text{ mm}$, a wall thickness $s = 3 \text{ mm}$ and a filling height of the liquid inside the tube of $h_f = 250 \text{ mm}$.

From Fig. 1 it can be seen that as long as the position of the interferometer laser remains perpendicular to the always-horizonal reflective surface of the liquid (the granite mounting of the interferometers helps to ensure this perpendicularity), a slight bending or tilting of the U-tube will have only a minimal effect on the result of the liquid height difference measurement. This minimal effect is further supported by the positioning of each interferometer's two laser beams within the neutral axis of the tube's bending line, as depicted in Fig. 1a.

The deformation of the tubes under pressure will result in an unsymmetrical displacement of the liquid level compared to the zero condition where the pressure in both tubes is equal. Nevertheless, after adding the two non-equal liquid displacement values, the resulting value of h can be used for the calculation of the pressure since the fundamental equation $p = \rho g h$ always remains valid.

Another influence on the length measurement is the deflection of the viewport window under the pressure load, or to be more precise, the change it experiences in the time between the zero condition and the measurement. This is the case when the mirror for the differential interferometer's reference beam is realized as a local metallic coating applied to the lower side of the viewport

window. Given that the pressure load inside the reference tube will not change during the measurement, the deflection of this viewport window is not crucial.

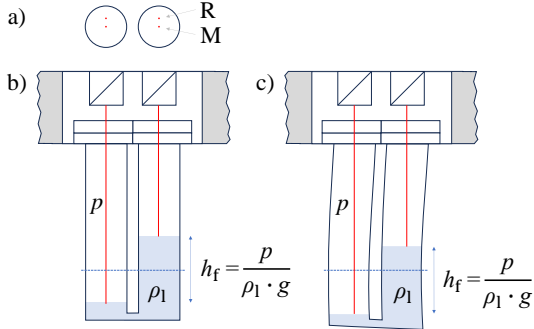


Fig. 1. a) Top view of the position of the two laser beams, the measurement beam M and the reference beam R, when entering the viewport window; b) change in height of the manometric liquid in response to the pressure change p from the level of the liquid when the pressure is equalised in both tubes (indicated by dashed line); c) like b, but with an exaggerated portrayal of the dimensional changes to both tubes under the material strain that builds up when pressure is applied to the left tube.

For a mounted boron-crown glass window with an open radius $r = 20$ mm, a thickness $t = 10$ mm, a Young's modulus $E_{gl} = 79.2$ GPa and an applied maximum pressure difference $p = 2$ kPa, the deflection d_w , allocated at the window's centre could be calculated as follows:

$$d_w(p) = \frac{0.171 \cdot r^4}{E_{gl} \cdot t^3} p = 0.7 \text{ nm}. \quad (2)$$

When transferring the result into the unit of pressure, with a liquid density of $\rho_1 = 830$ kg/m³ and the gravitational acceleration $g = 9.81$ m/s², and assuming a rectangular distribution, the relative standard uncertainty due to the deflection of the window $u(p_{d_w})/p$ can be calculated:

$$\frac{u(p_{d_w})}{p} = \frac{\rho_1 g d_w}{2\sqrt{3} \cdot p} = 0.9 \times 10^{-9}. \quad (3)$$

The calculation in Eq. 3 is still a conservative estimation of the error as the mirroring surface of the LCM is located 12.5 mm away from the centre of the viewport window. The true result will therefore always be smaller than yielded here. Nevertheless, the contribution of the deflection of the viewport window to the pressure uncertainty is negligibly small.

A concept for the positioning of the reference mirror that makes the length measurement even less dependent on pressure is shown in Fig. 2. Here, the reflective surface is located next to the underside of the viewport's window but is not coupled to it.

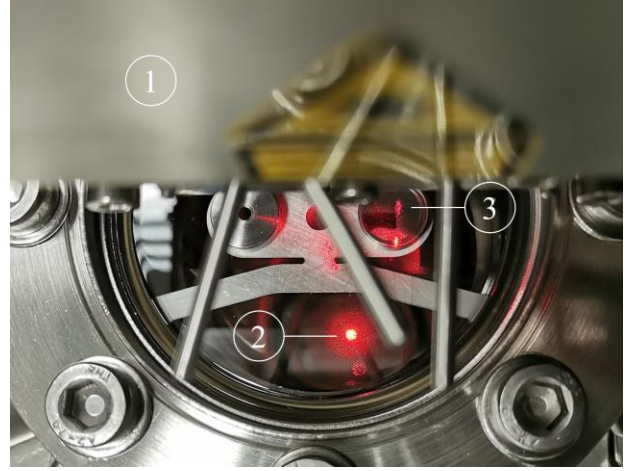


Fig. 2. Top view of one tube of the LCM where the two beams of the differential interferometer (1: partial view, perspective blurred) enter the tube. The measurement beam is orientated along the tube's centreline and its reflections from the pool float and the liquid's surface are clearly visible. The reference beam is arranged 12 mm offset to the measurement beam and is reflected from a mirroring surface (3) that is positioned below the viewport's window.

Depending on the measurement mode of the LCM, the measurement beam of the interferometer passes downward towards the liquid surface through the pressure transmitting medium, usually nitrogen gas in the pressure range of (0 to 2) kPa for absolute pressure measurements and (100 to 102) kPa for gauge pressure measurements. The refractive index [4] of this medium needs to be factored into the liquid column displacement calculation. An accurate refractive index calculation is possible as the parameters of temperature and pressure are well known, due to the stabilisation of the entire setup by a thermostatic bath, and given the fact that inside a pressure standard like an LCM, even without a refractive index correction, the pressure uncertainty lies in the order of a few pascals.

Inside each tube, the free surface of the liquid is used as the laser beam reflecting mirror [5, 6, 7]. This is advantageous as floats on the liquid surface carrying a mirror or retroreflector give poor length signal reproducibility following any movement of the liquid column due to unstable wetting conditions at the float/liquid interface. Even with the low reflectivity of the liquid of only 4%, measurement is still quite feasible using the interferometers described here.

To prevent interferometer signal losses caused by the formation of waves that build up at the contact line between the liquid and the inner tube wall, especially when the liquid column moves, an open pool float made of polytetrafluoroethylene (PTFE) is positioned on the liquid surface inside each tube of the LCM [6, 7]. The pool wall's inclination was chosen in a way to support the building of

a horizontal contact angle between the liquid and the pool wall to minimise contact angle effects and hence support the formation of a free liquid surface inside the pool. In addition, damping effects in the shallow pool of adjustable depth inhibit the formation of surface waves.

III. SUMMARY

This paper presents the length measurement system of the liquid column manometer found at PTB. Its key features – the use of differential interferometers and the direct sensing of the manometric liquid's nearly free surface aided by pool floats – offer a sound basis for the realisation of a robust and accurate length measurement system.

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