

High-Resolution Displacement Measurement using a Femtosecond Frequency Comb

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Abstract:

The work introduces a high-resolution displacement measurement method in a nanometer scale using passive Fabry-Perot cavity. The optical frequency of selected resonant mode of the cavity is monitored during changes of the cavity length. It uses an optical mixing the frequency of a tunable laser with frequency teethes of a femtosecond optical synthesizer (optical frequency comb). The tunable laser diode is locked to a selected mode of the optical resonator to monitor the exact mirror distance of Fabry-Perot cavity. Then a laser diode optical frequency is mixed with the beam from the stabilized femtosecond frequency comb. Due to wide spectrum of the comb and worse signal-to-noise ratio of a beat-note signal a fiber Bragg grating is used for narrow-band selection of desired components of the femtosecond comb. The beat-note signal is processed by a fast photo detector and frequency counter. The displacement of the movable mirror of the Fabry-Perot cavity is calculated on basis of the beat-note signal. The main advantage is in linear course of the scale due to using the Fabry-Perot cavity as measuring device.

Introduction

The significant demand comes from a branch of metrological Atomic Force Microscopes (mAFM) where precise measurement of displacement of cantilever in a Z-axis is needed [1].

Research teams in the world works on various measuring methods based on a laser interferometry. But the most precise systems use an optical resonator as a measuring device. The idea is: the cantilever of mAFM is equipped by a micro mirror which makes an optical cavity with the second mirror placed inside the frame of the mAFM. A sweeping of the cantilever during the scanning process makes changes of the cavity length.

On the other hand the detection of the optical resonator length is very difficult. It leads to optical frequency method where the frequency of selected resonant mode is monitored during changes of the cavity length. The measurement of this optical frequency in the past was done with tunable laser locked to this mode and mixing with an optical frequency

standard. But a bandwidth of the fast photodetector and frequency counter (measuring beat-note) limited the displacement range of measurement [2].

Theory of the length measurement with Fabry-Perot cavity and femtosecond comb

The method employing the optical frequency comb is based on the femtosecond laser and Fabry-Perot optical cavity. The optical frequency comb produces periodically a train of femtosecond pulses. The train of pulses is characterized by the central wavelength, period of pulses, and their shape and by pulse to pulse phase shift. It produces an optical frequency spectrum of comb lines around the central optical frequency (wavelength). The frequency f_i of a comb spectral component is represented by expression:

$$f_i = f_{ceo} + i \cdot f_{rep} \quad (1)$$

where i is the number of a comb line (typically in the order of 10^6) and f_{rep} , and f_{ceo} are frequencies typically set in radio frequency (RF) domain, called repetition and offset frequency, respectively. The repetition frequency f_{rep} is indirectly proportional to the period of the train pulses and the offset frequency f_{ceo} is caused by phase-shifts between envelope of the pulses and their carrier wavelength [3]. Relative stability of spectral lines of free running femtosecond comb as well as relative stability of frequencies (f_{rep} and f_{ceo}) reach the value of 10^{-7} . It is caused by the thermal dilatation of the body of the laser cavity. Therefore a precise stabilization of f_{rep} and f_{ceo} frequencies is necessary for more stable frequency comb spectrum. The stability of i -th optical component (spectral line) of the comb depends multiplicatively (relative stability remains the same) on the stability of the repetition frequency f_{rep} based on Eq. (1) and additively on the offset frequency f_{ceo} . The repetition frequency is very intensively present in RF spectrum and thus it is easily detected and phase locked to a multiple of an atomic clocks RF signal (typically 10 MHz) by phase-lock servo-loop [3]. The offset frequency has to be retrieved from comb spectrum by self-referencing method called as the f - $2f$ interferometric technique. Approximately 100 nm narrow spectrum of the femtosecond laser (with central wavelength $\lambda = 1541$ nm in our case) is broadened by this method to the octave wide spectrum (in our case an interval from 1050 nm to 2100 nm) by a four-wave mixing technique in a photonics crystal fibre. Longer wavelengths (frequency f_i) are frequency doubled by non-linear crystal to shorter wavelengths (resulted frequency $2f_i$). Doubled longer wavelengths are optically mixed with short wavelengths (frequency f_{2i}) producing the offset frequency:

$$2f_i - f_{2i} = f_{ceo} \quad (2)$$

The offset frequency f_{ceo} is as well as the repetition frequency f_{rep} phase locked to the stable oscillator or atomic clocks RF signal by a servo-loop. The result is the optical comb frequency spectrum with relative stability of all of frequency components coupled to the relative stability of the RF oscillator or atomic clocks (thus more than 10^{-14}).

The optical resonator FPC consists of two mirrors separated by the geometrical distance L_{cav} . This length is represented by the cavity spacer. The optical resonator is an instrument which transfers in resonance any precise frequency fulfilling the relation:

$$\nu = m \cdot \nu_{FSR}, \quad (3a)$$

where m -th is a longitudinal mode of the FPC and where ν_{FSR} is the free spectral range (FSR) of FPC characterized by the optical distance between mirrors:

$$\nu = \frac{c}{2nL_{cav}} \quad (3b)$$

and in a confocal FPC geometry:

$$\nu = \frac{c}{4nL_{cav}}, \quad (3c)$$

where c is speed of the light, n is the refractive index of air inside the cavity. The optical frequency of m -th longitudinal mode transmitted through the con-focal FPC can be expressed then:

$$f_m = \frac{mc}{4nL_{cav}}. \quad (4)$$

The comb spectrum is transmitted through FPC if some of optical frequency spectral lines f_i (Eq. (1)) meet the resonance frequency from Eq. (4). Reasonable number of lines transmit only than FRS of the FPC is an integer multiple of f_{rep} thus: $\nu_{FSR} = x \cdot f_{rep}$ [4]. The optical signal at the output of the FPC depends on the distance between mirrors in FPC. Then the optical output of the FPC contains number of separated resonance lines [5]. Otherwise the optical output from FPC represents the similar signal to the white light continuum and the signal doesn't contain any resonance peaks.

These problems can be solved by the use of a single-mode stable laser with the one resonance peak per FSR of the cavity. Because the FPC's FSR is fixed due to the stable

spacer a tunable laser source searches for the peak resonance. The use of tunable laser source is advantageous for following the small peak changes. Optical frequency of the closest optical frequency comb line is used as a precise frequency reference to the tunable laser source. The optical frequency of the tunable laser referenced to optical comb line (Eq. (1)):

$$\nu_{opt} = f_{ceo} + i \cdot f_{rep} \pm f_{beat} \quad (5)$$

where f_{beat} is the beat frequency between the closest optical comb spectral line and the tunable laser locked to the FPC. The beat frequency could be of both polarities and this polarity is easily evaluated from the measurement results with respect to temperature length variations or for fixed cavity length from variations of beat frequency f_{beat} with respect to the offset frequency f_{ceo} . Let us equal f_m to ν_{opt} , then:

$$\nu_{opt} = f_m = f_{ceo} + i \cdot f_{rep} \pm f_{beat} = \frac{mc}{4nL_{cav}} \quad (6)$$

Extracting the exact length of the FPC from Eq. (6) then:

$$L_{cav} = \frac{mc}{4n(f_{ceo} + i \cdot f_{rep} \pm f_{beat})} \quad (7)$$

Because i and m are integer constants and c is exact speed of the light in vacuum the exact length L_{cav} of FPC is determined by frequencies f_{ceo} , f_{rep} and f_{beat} . Repetition and offset frequencies are controlled by RF reference by f-to-2f stabilization. The influence of refractive index of air on the result is suppressed when the FPC is inserted into the vacuum chamber where refractive index equals 1.0000:

$$L_{cav} = \frac{mc}{4(f_{ceo} + i \cdot f_{rep} \pm f_{beat})} \quad (8)$$

The FPC mirror distance is then defined by f_{beat} frequency between optical frequency comb and the tunable laser.

The optical set-up and proposal of the equipment

The DFB laser diode works at central wavelength of 1542.14 nm (194.40 THz) and can be tuned by the operating temperature control for more than 2 nm (250 GHz). Another temperature control within the range from -10°C to 80°C was used for tuning of the FBG's central selective wavelength. It ensures tuning of the central wavelength 1540.045 nm in the interval ± 1.1 nm. The FBG has 0.186 nm FWHM and is used to increase SNR during beat-note measurement. A laser reference to DFB laser diode is represented by Er:doped stabilized femtosecond modelocked laser with 100 MHz repetition frequency and central wavelength at 1550 nm. The laser is the heart of the optical frequency comb. The 1500-1600 nm spectral output of the system as well as laser itself are fiber-based. The optical frequencies of the femtosecond mode-locked laser are stabilized by f-2f interferometry to 10 MHz RF temperature stabilized quartz oscillator with long term stability reference to Cs clock through GPS receiver.

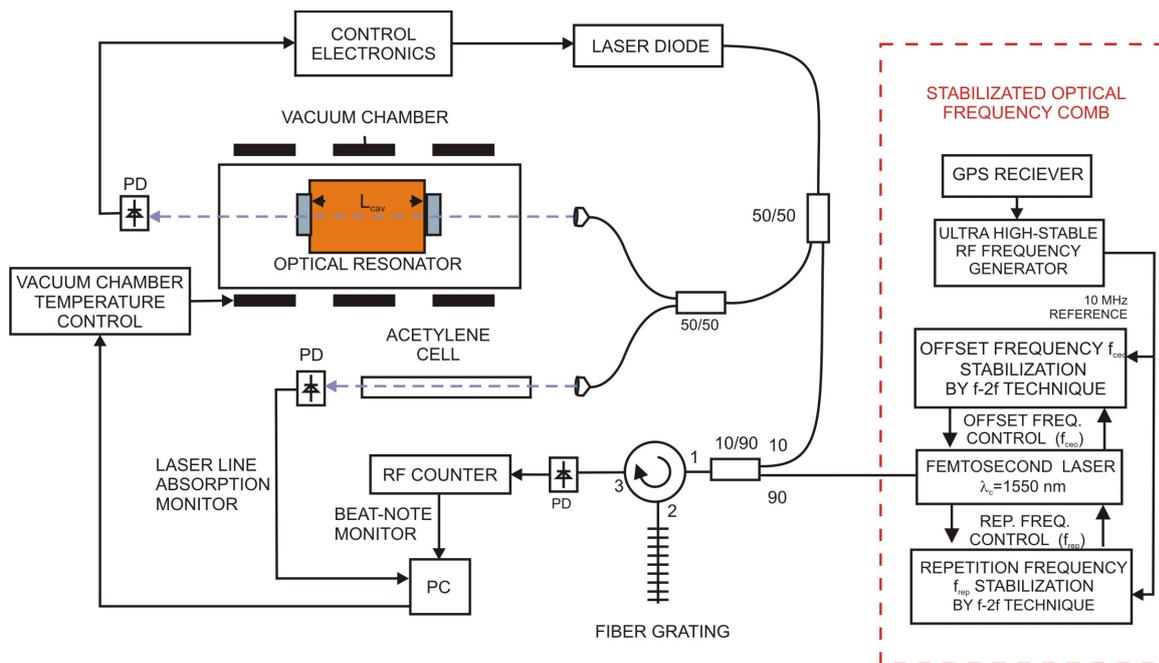


Fig. 1 Scheme of the optical setup for displacement measurement by optical resonator using stabilized optical frequency comb and additional laser diode.

Figure 1 presents the scheme of the set-up with DFB diode laser coupled to the fiber. The DFB laser output is splitted into two 50% parts by a 50/50 coupler. 10% of one output is mixed with 90% of the output from Er:doped mode-locked fiber laser comb into 90/10 fiber coupler. The 90% of optical frequency comb power interferes with 10% of the DFB laser. Interference signal is coupled to the input port (1) of a fiber circulator. The light in fiber

circulator travels from port 1 to port 2 and from port 2 to port 3 whereas the back reflection from port 2 to port 1, from port 3 to port 2 as well as from port 3 to port 1 is not possible. The frequency comb components within the range of 0.186 nm (FWHM of FBG) interferes with single frequency of DFB laser. Beat frequency between the comb components and the DFB laser diode are detected by infrared fast photodetector (PD), counted by a RF counter and recorded by the computer.

Second output of 50/50 coupler from DFB laser output light is splitted in 50/50 coupler to 50% to the acetylene cell and 50% to the FPC. Infrared photo detectors PDs monitor absorption at the acetylene cell and the output resonance mode spectrum of the FPC.

Experimental testing and conclusion

The experiment has been performed in a dynamic regime where one of the FPC mirror is positioned by piezoelectric transducer (PZT). The range of displacement of the PZT was 700 nm approx. The first experimental result recorded by the equipment is shown in Figure 2. The PZT was tuned by a high voltage to maximum elongation and back to the starting position. The tuning speed was 0.9 nm / s,

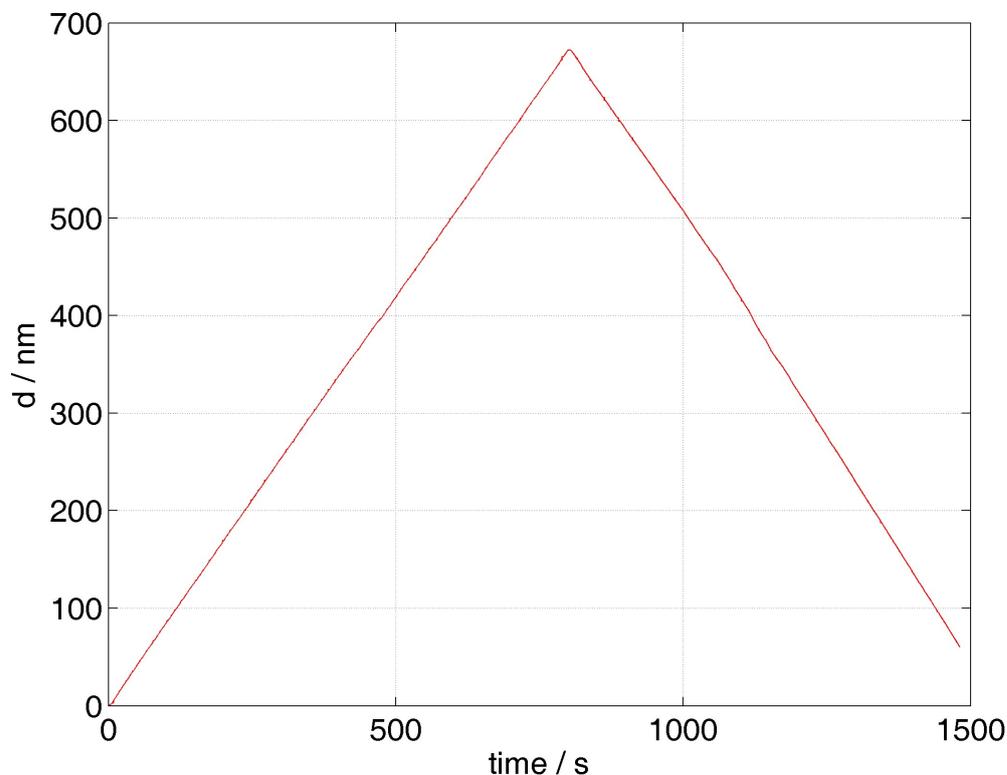


Fig. 2 The first record of the tuneability of the FPC by the PZT transducer up to 700 nm elongation.

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References

- [1] Dai G, Pohlenz F, Min Xu, Koenders L, Danzebrink H U and Wilkening G (2006) Accurate and traceable measurement of nano- and microstructures, *Meas. Sci. Technol.*, 17: 545–552.
- [2] Číp O, Petrů F, Buchta Z and Lazar J (2007) Small displacement measurements with subatomic resolution by beat frequency measurements. *Meas. Sci. Technol.*, 18: 2005-2013
- [3] Udem T, Holzwarth R, Zimmermann M, et al (2004) Optical frequency-comb generation and high-resolution laser spectroscopy. *Topics in Applied Physics*, 95: 295-316
- [4] Telle H R, Steinmeyer G, Dunlop A E, Stenger J, Sutter D H and Keller U (1999) Carrier-envelope offset phase control: A novel concept for absolute optical frequency measurement and ultrashort pulse generation. *Applied Physics B*, 69:327-32
- [5] Šmíd R, Číp O, Čížek M, Mikel B and Lazar (2010) Conversion of stability of femtosecond mode-locked laser to optical cavity length. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control* 57(3): 636-40