Virtual Instrument for the Estimation of the Time Delay Using Conditional Averaging of Random Signals

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Abstract **–** The paper presents a virtual instrument (*VI*) which allows to carry out the simulation and experimental studies of the various statistical methods of random signal time delay measurement including the methods developed by the authors which employ the conditional averaging of signals. The *VI* includes generators and oscilloscope and also a PC with the DAQ card and the software developed in the LabVIEW environment. The instrument allows to model and generate the stochastic signals with definite statistical parameters, to acquire signals from external sources, and to perform the time delay estimation with the use of several methods. This study describes the principle and properties of the method which employs the conditional average value of the absolute value of delayed signal (*CAAV*), and some exemplary results of this method with the use of the built instrument are presented. The results obtained with the *CAAV* method have been compared with the results obtained by the best known and used at the measurements of random signal time delay method of cross correlation (*CCF*). It has been found that the *CAAV* method is featured with lower time delay variance than the *CCF* within the signal-to-noise ratio higher than 0.35.

I. Introduction

The estimation of the time delay (*TDE)* is the frequently analyzed issue at signal processing, including the radar technique, radio astronomy, locating of noise transferring ways, or non-contact measurements of the parameters of media flow (eg. two-phase flows). Statistical methods are commonly used at the determination of the time delay of the stochastic signals obtained from two or more sensors. This issue is widely presented in publications [1-6] where a number of methods based on the signal analysis in the time and frequency domain are described. Among the conventional methods for stationary signals one of the best known is the cross-correlation function (*CCF*) in the time domain, and the phase of the power spectral density in the frequency domain [2,6-8]. Under specific conditions also difference methods [5] can be used, or the relatively little known methods based on the conditional averaging of signals [9-12]. At the studies of the features of the statistic methods for the measurement of time delays pretty often virtual instruments (*VI)* are used, and the commercial solutions are based on signal processors or personal computers with appropriate software [13]. In this paper the *VI* has been described, which has been built in order to carry out the pertinent researches, and some its capabilities have been described illustrated with an example of the investigation of the *TDE* method using the conditional average value of the absolute value of delayed signal (*CAAV*) with reference to the correlation method.

II. The model of the measurement of signals and the principle of *TDE* using *CAAV*

In many issues related to the estimation of the time delay, eg. the measurements of the parameters of the transport and flow of solids, the relationship between the $y(t)$ and $z(t)$ signals obtained from two different sensors can be presented as [2]:

$$z(t) = kx(t-\tau_0) + n(t) = y(t) + n(t) \qquad (1)$$

where: $x(t)$ – stationary random signal with the normal probability distribution $N(0, \sigma_x)$ and definite correlation-spectrum characteristics; k – constant; τ_0 – transport delay equal to the quotient of the distance L between the sensors, and the object average velocity v; $n(t)$ – stationary white noise non-correlated with the signal $x(t)$ with the $N(0, \sigma_z)$ distribution. In practice the $y(t)$ and $z(t)$ signals are the realisation of the ergodic $X(t)$ and $Z(t)$ processes.

The *CAAV* function is defined as follows:

$$A_{|z|}(\tau) = E\left\{ |z(t+\tau)| \big|_{x(t)=0} \right\} = \int_0^{\infty} |z| \cdot p\left(|z(t+\tau)| \big|_{x(t)=0} \right) dz \qquad (2)$$

where: *E[ ]* stands for the operation of the expected value; T - averaging time; τ - transport delay; $p\left(|z(t+\tau)| \big|_{x(t)=0}\right)$ - the conditional probability density for the $z(t+\tau)$ signal value for the $x(t) = 0$ condition.

The principle of the determination of the τ_0 transport delay consists in defining the position of the main minimum of the *CAAV* function (Fig. 1).

a) b)

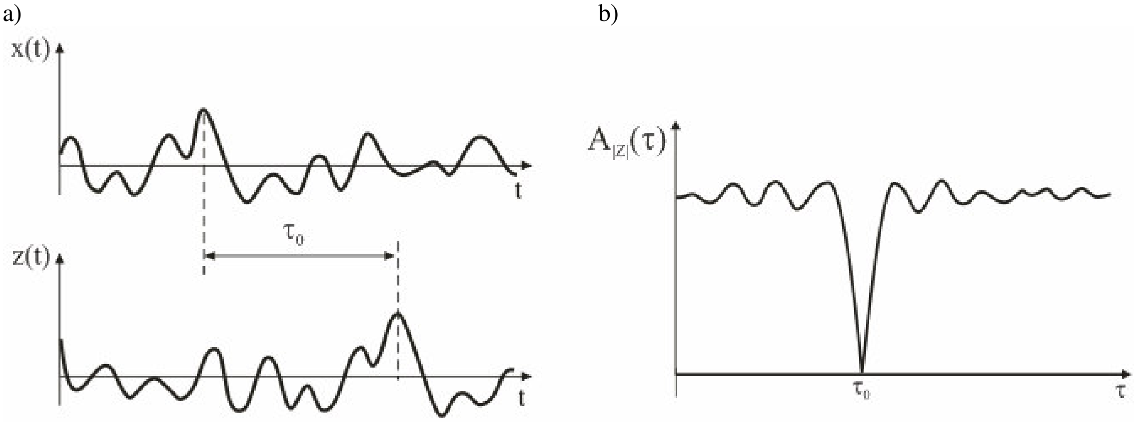


Fig. 1. The principle of the estimation of the time delay using *CAAV* $A_{|z|}(\tau)$: mutual delayed stochastic signals (a); *CAAV* functions for $n(t)$=0 (b)

III. Virtual Instrument

Within the study of the methods for the stochastic signal analysis, including the statistic methods of the time delay measurement, the *VI* has been built in the Department of Metrology and Diagnostic Systems of the Rzeszów University of Technology. The block diagram of the *VI* is presented in Fig. 2. The target purpose for this *VI* is:

- computer modelling of random processes with initially set characteristics (including those mutually delayed);
- multi-channel physical generation of stochastic voltage signals on the basis of the developed models. These signals will be used as coercive ones, eg. at the identification and studies of signal and system properties;
- synchronous multi-channel acquisition of the measurement signals coming from the investigated objects (including but not limited to the workstations for the investigations of two-phase liquid-solid flow processes);
- both on-line and off-line analyses of signals with the use of the studied methods employing among other things the conditional signal averaging.

The *VI* comprises:

- two functional Tektronix AFG 3102 generators,
- digital Tektronix TDS 2002 oscilloscope,
- the system for data modelling and acquisition (PC with PCI NI-6143, PCI-GPIB cards and software).

The control-measurement software developed in the LabVIEW environment consists of the modules carrying out the mentioned task. The use of the NI PCI-6143 card allows to carry out a synchronous acquisition and recording in eight channels with the maximum sampling frequency 250 kHz. The recorded signals are saved in a file to archive them and possible further analysing in the post-processing procedure. The target aim is to transfer the signals coming from the investigated objects to the NI 6143 card inputs in parallel with the coercion acts from generators (system and arrangement identification), or with disconnected generators (two-phase flow investigation).

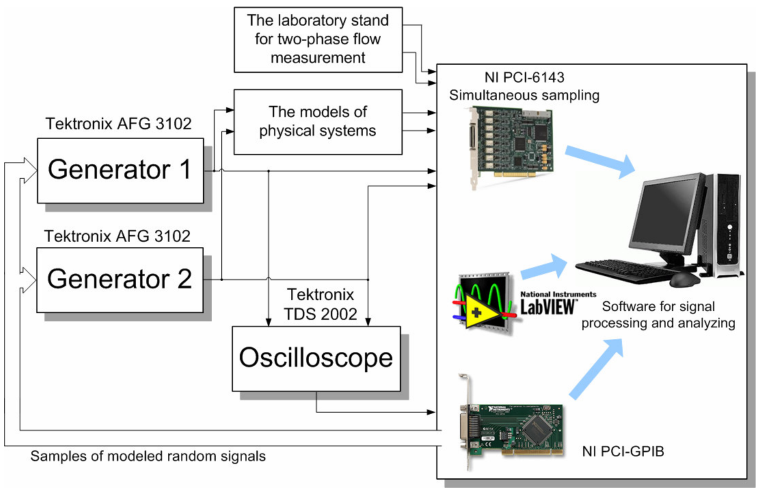
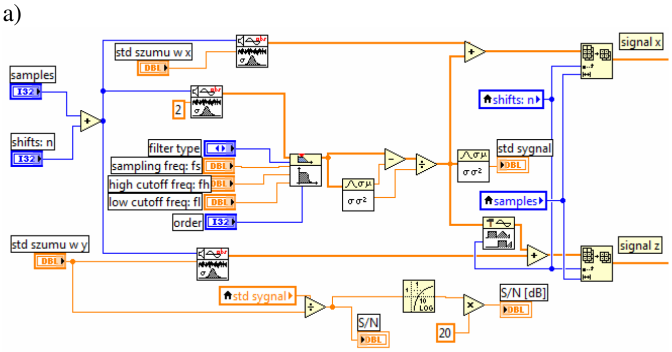


Fig. 2. Block diagram of the VI

The application developed in the LabVIEW environment allows to control all the capabilities of the *VI* mentioned above. In case of the investigation of the methods of the time delay measurement the user selects the experiment type: either for real data or computer simulations (Fig 3a). In both cases the program receives two tables containing the instantaneous values of the analyzed signals. In case of measurements the data can be analyzed in post-processing. ie. the measurement result are first recorded, and then the program takes readings from appropriate files (Fig. 3b).

a)



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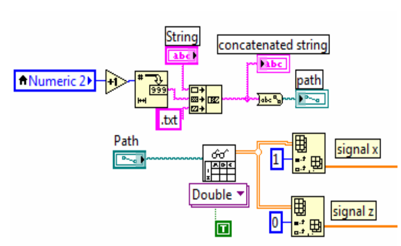
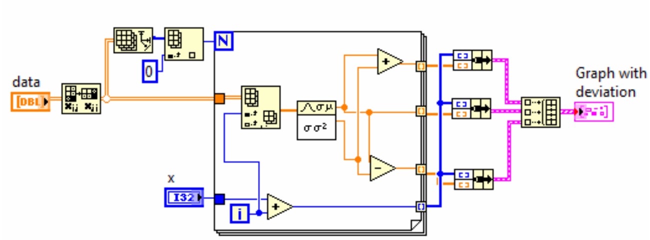


Fig. 3. Parts of the VI diagram: computer simulations (a); taking readings of the measurement data (b)

The experimental results are presented in both numerical values and graphs. To make the comparison of the obtained results possible the determination of the normalized standard deviations of the *CAAV* and *CCF* characteristics has been carried out with a specially developed subprogram (sub-*VI*). The diagram and the icon of this subprogram is presented in Fig. 4.

a)



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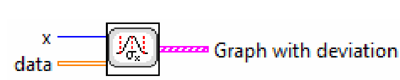

Fig. 4. The diagram (a) and the icon (b) of the subprogram used for the calculation of the normalized standard deviations of the investigated characteristics

IV. Some results

The discrete *CAAV* estimator can be presented by the formula:

$$\hat{A}_{|z|}(l) = \overline{|z(l)|}\Big|_{x(n)=0} = \frac{1}{M}\sum_{n=1}^{M}|z(n+l)|\Big|_{x(n)=0} \tag{3}$$

and the discrete *CCF* estimator is determined from the relationship:

$$\hat{R}_{xz}(l) = \frac{1}{N}\sum_{n=1}^{N} x(n)z(n+l) \tag{4}$$

where: N - the number of samples; $l = \tau/\Delta t$, $l = 0,1,2...L-1$; $n = t/\Delta t$, Δt – sampling interval, M – the number of passes through the x signal zero.
In Fig. 5 some exemplary courses of the *CAAV* and *CCF* estimators are presented, obtained in the *VI* described above, by the simulation process for the discrete signal models (1) and the parameters: $M = N = 3000$, $l_0 = \tau_0/\Delta t = 100$; $k = 1$, $\sigma_x = 1$, $\sigma_n = 0$.

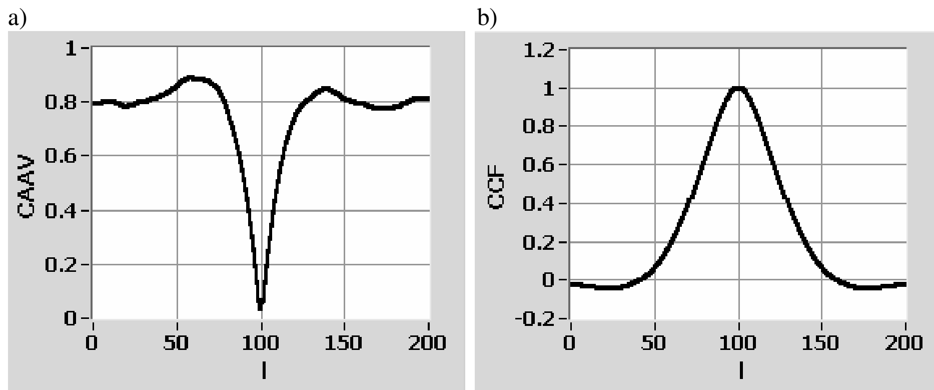

Fig. 5. *CAAV* (a) and *CCF* (b) obtained from the simulation for $\sigma_n = 0$

Within the investigation the standard deviations of the *CAAV* and *CCF* functions are analyzed and compared in the extreme values points of these functions, as well as analyzed and compared are the delay standard deviations. For the N of the non-correlated sample pairs the comparison of the theoretical relative standard deviations of the estimators (3) and (4) leads to the following relationship [12]:

$$\frac{\varepsilon\left[\hat{A}_{|z|}(\tau_0)\right]}{\varepsilon\left[\hat{R}_{xz}(\tau_0)\right]} = \frac{\dfrac{SD\left[\hat{R}_{xz}(\tau_0)\right]}{R_{xz}(\tau_0)}}{\dfrac{SD\left[\hat{A}_{|z|}(\tau_0)\right]}{A_{|z|}(\tau)_{max}}} \approx \left[\frac{N}{M}\,\frac{\left(\dfrac{\pi}{2}-1\right)\left(\dfrac{1}{1+k^2 SNR}\right)}{\left(2+\dfrac{1}{k^2 SNR}\right)}\right]^{1/2} \tag{5}$$

where $SNR = (\sigma_x/\sigma_n)^2$ is the signal-to-noise ratio.
The plots of the relationship (5) for $k=1$ and two values of the N/M ratio is presented in Fig. 6.
The simulation experiment consisted in the multi-repeated simulation and determination of the relative experimental standard deviations for the characteristics being calculated within the neighbourhood of the extreme points according to the formulas:

$$\hat{\varepsilon}\left[\hat{A}_{|z|}(l)\right] = \left[\frac{1}{K}\sum_{i=1}^{K}\left(\frac{\hat{A}_{|z|}(l)_i - \overline{\hat{A}_{|z|}(l)}}{\left[\hat{A}_{|z|}(l)_i\right]_{max}}\right)^2\right]^{1/2} \tag{6}$$

$$\hat{\varepsilon}\left[\hat{R}_{xz}(l)\right] = \left[\frac{1}{K}\sum_{i=1}^{K}\left(\frac{\hat{R}_{xz}(l)_i - \overline{\hat{R}_{xz}(l)}}{\left[\hat{R}_{xz}(l)_i\right]_{max}}\right)^2\right]^{1/2} \tag{7}$$

where: K – the number of experiment repetitions.
In Fig. 6 the comparison of the experiment results obtained for the two N/M ratio values at $K=1,000$ with the calculation results is presented.

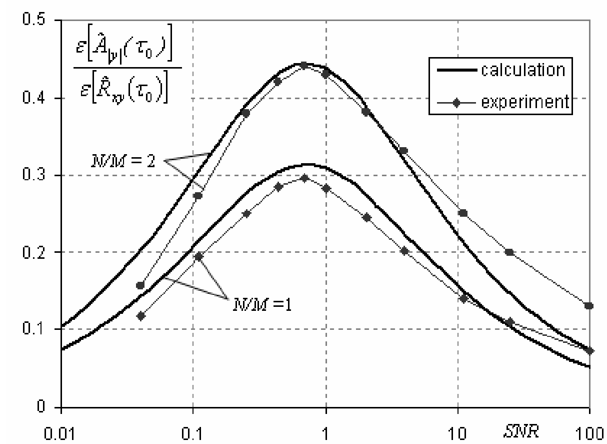

Fig. 6. The plots of the relations (5) and (6)/(7) for N/M = 1 and N/M = 2

Fig. 7 presents some exemplary *CAAF* and *CCF* curves within the neighborhood of the extreme points with the range of one relative standard deviation $\hat{\varepsilon}$ marked, determined according to the (6) and (7) formulas, respectively, for K= 1,000 and SNR = 1.

a) b)

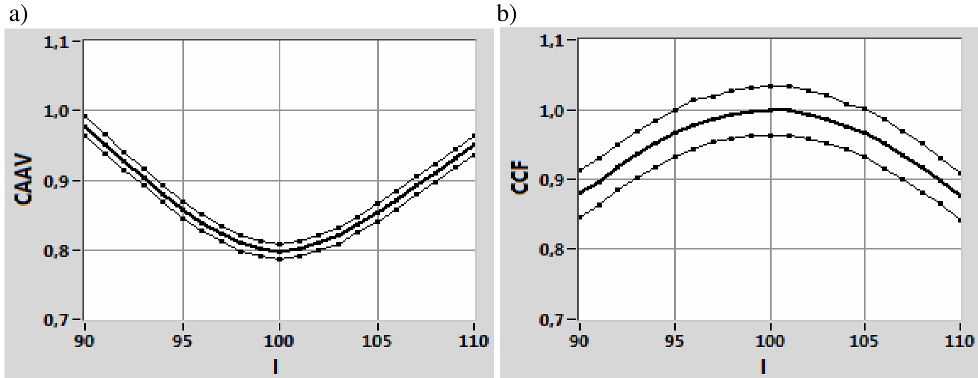

Fig. 7. *CAAV* (a) and *CCF* (b) for *SNR* = 1

The analysis of the theoretical relationships determining the standard deviations of the transport delay estimator evaluated on the basis of the *CAAV* and *CCF* [8, 10] has allowed to obtain the formula:

$$\frac{SD[\hat{\tau}_0]_{CAAV}}{SD[\hat{\tau}_0]_{CCF}} \approx \left[\frac{N\left(\frac{\pi}{2}-1\right)}{M\ k^2 SNR\left(2k^2 SNR+1\right)} \right]^{1/2} \tag{8}$$

The curves illustrating the relationship (8) for k=1 and two N/M ratio values are presented in Fig. 8.
In the conducted simulation experiment, like before, the determination of the *CAAF* and *CCF* was repeated many times, and the transport delay estimator was determined at each repetition. Then the experimental standard deviations of the $\hat{S}D[\hat{\tau}_0]_{CAAV}$ and $\hat{S}D[\hat{\tau}_0]_{CCF}$ delay estimators were determined. The results obtained for two N/M ratio values at K= 1,000 have been compared with the calculations which is shown in Fig. 8.