UNCERTAINTY ANALYSIS OF THE ADC HISTOGRAM TEST USING TRIANGULAR STIMULUS SIGNALS

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Abstract-This paper addresses the uncertainty of the estimates of ADC testing obtained with the Histogram Method when a triangular stimulus is used. Expressions are presented for the computation of the standard deviation of the transition voltages and code bin widths. These can be used for the Ramp Vernier Test which is a novel test method which will be included in the new version of the IEEE 1057 standard currently in balloting.

I. Introduction

There are several test methods for the characterization of analog to digital converters (ADCs). One of the most widely used is the Histogram Test Method which allows the estimation of an ADC transfer function. Traditionally it is used with a sinusoidal stimulus signal to dynamically characterize the converter [1-5]. Recently this procedure has been employed with triangular waves to static test ADCs with a method called "Ramp Vernier" [6-9].

As with any measurement, estimating the transitions voltages and code bin widths of an ADC requires also an estimative of its uncertainty. This paper contributes to this in the special case of triangular stimulus signal and an Histogram procedure. In the past, the authors have studied the bias of this test method and shown how much overdrive to use in order to minimize the effects of additive noise on the error of the transition voltages and code bin widths [10].

In section II we describe the Histogram test method and introduce the variables that are going to be used to describe the ADC transfer function and stimulus signal. In section III we compute the variance of the transition voltages and in IV the variance of the code bin widths. Finally in section V we draw some conclusions.

II. Histogram Test Method

The Histogram procedure consists in using the ADC under test to acquire a large number of samples of a known signal. The digital codes of the samples are then used to compute a cumulative histogram ($CH[k]$) which is the number of samples with a code equal to or lower than each of the possible ADC output codes k. It is an array of 2^N values, where N is the number of bits of the ADC. From this array, the ADC transition voltages are estimated using

$$T[k] = 2A\frac{CH[k-1]}{M} - A \quad , \quad k = 1 \ldots 2^N - 1 , \qquad (1)$$

and the code bin widths using

$$W[k] = 2A\frac{H[k]}{M} \quad , \quad k = 1 \ldots 2^N - 2 , \qquad (2)$$

where M is the number of samples and $H[k]$ is the number counts of the histogram which is the number of samples with each of the possible ADC outpout codes k. The relation between the value of the histogram and the cumulative histogram is

$$H[k] = CH[k] - CH[k-1] \quad , \quad k = 1 \ldots 2^N - 1 . \qquad (3)$$

In the traditional histogram method, a full-scale periodic stimulus signal is applied to the ADC and a certain number of samples are acquired at a constant rate asynchronously with the stimulus signal. The value of the ADC transition voltages is obtained by comparing the number of samples obtained in each code bin with the number expected in the case of an ideal ADC. To determine this last number it is necessary to know the probability distribution of the sample voltages, that is, the value of the input signal at the instant of sampling.

Consider each sample j (j = 0, 1, …, $M-1$) ideally acquired at instant t_j. Without loss of generality the time origin can be set to the ideal sampling instant of the first sample ($t_0 = 0$). The phase of the samples (γ_j), relative to the stimulus signal of frequency f, is thus

$$\gamma_j = 2\pi f \cdot t_j + \varphi \tag{4}$$

where φ represents the phase of the stimulus signal at the ideal instant of acquisition of the first sample. The value of the sinusoidal stimulus signal in the sampling instant of sample j can be written as

$$x_j = d - A \cdot \operatorname{tri}\left(2\pi f \cdot t_j + \varphi\right) \tag{5}$$

where d and A are the stimulus signal offset and amplitude. The "tri(x)" function represents a general triangular wave function spanning from $-\pi$ to π in x and from -1 to 1 in amplitude:

$$\operatorname{tri}(x) = \begin{cases} 1 - 4\left\langle \dfrac{x}{2\pi} \right\rangle & , 0 < \left\langle \dfrac{x}{2\pi} \right\rangle \le \dfrac{1}{2} \\ 4\left\langle \dfrac{x}{2\pi} \right\rangle - 3 & , \dfrac{1}{2} < \left\langle \dfrac{x}{2\pi} \right\rangle < 1. \end{cases} \tag{6}$$

The values of the sampled voltages (v_j) are equal to the value of the stimulus signal in the sampling instant (x_j) plus the input-equivalent wideband noise (n_v).

$$v_j = n_v + d - A \cdot \operatorname{tri}\left(2\pi f \cdot t_j + \varphi\right) \tag{7}$$

To simplify the computations, some normalizations are made. Let u_j be the normalized sample voltage.

$$u_j = \frac{v_j - d}{A} = \frac{n_v}{A} - \operatorname{tri}\left(\gamma_j\right) = n - \operatorname{tri}\left(\gamma_j\right) \tag{8}$$

where n is the normalized input-equivalent wideband noise and γ_j is the sample phase.

III. Variance of the Transition Voltages

A. General Sampling

In this section the determination of the probability distribution of the number of counts of the histogram is presented for a general type of sampling (random, synchronous or asynchronous). In the next section the particular case of asynchronous sampling will be considered.

Considering also that the input-equivalent noise is normally distributed, with a null mean and a standard deviation σ_n, the probability density function (p.d.f.) of the sample voltages (u_j) is

$$f_{u_j}\left(u \mid \gamma_j\right) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-j\frac{\left(u + tri\left(\gamma_j\right)\right)^2}{2\sigma_n^2}} . \tag{9}$$

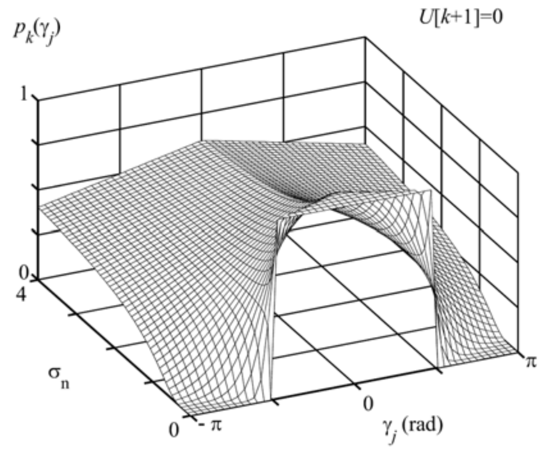

Fig. 1 Representation of the probability that a sample belongs to a class of the cumulative histogram as a function of the sample phase (γ_j) and of the input-equivalent noise standard deviation (σ_n). Example with $U[k+1]$=0 (transition voltage equal to the DC value of the stimulus signal).

The probability that a sample j belongs to a class k of the cumulative histogram (p_k) is equal to the probability that the normalized sample voltage is between normalized transition voltage $U[k]$ and $U[k+1]$:

$$p_k(\gamma_j) = \int_{-\infty}^{U[k+1]} f_{u_j}(u \mid \gamma_j) \cdot du = \frac{1}{2} + \frac{1}{2} erf\left(\frac{U[k+1] + tri(\gamma_j)}{\sqrt{2} \cdot \sigma_n} \right). \tag{10}$$

This probability is depicted in Fig. 1 as a function of the sample phase and input equivalent noise standard deviation.

Consider now a variable w_k that takes the value 1 if a sample belongs to class k of the cumulative histogram and 0 if not.

$$f_{w_k}(w \mid \gamma_j) = \begin{cases} p_k(\gamma_j) & , \quad w = 1 \\ 1 - p_k(\gamma_j) & , \quad w = 0 \end{cases} \tag{11}$$

This variable has a binomial distribution with mean p_k and variance $p_k \cdot (1 - p_k)$.

The number of counts in class k of the histogram (c_k) is the sum of variable w_k for all the samples. The p.d.f. of c_k is the convolution of the p.d.f. of the M variables w_k, because they are independent.

$$f_{c_k}(c \mid \varphi) = f_{w_k}(w \mid \gamma_0) * \ldots * f_{w_k}(w \mid \gamma_{M-1})(c) \tag{12}$$

Note that, now, the p.d.f. of the number of counts is conditional to the initial phase φ and not to the sampling instant γ_j. The conditional mean of c_k is the sum of the conditional means of the variables w_k for each sample. The same is true for the conditional variance.

The total p.d.f. of the number of counts can be obtained by integrating the product of the conditional p.d.f. with the p.d.f. of the initial stimulus signal phase (f_φ) [11].

$$f_{c_k}(c) = \int_{-\infty}^{\infty} f_{c_k}(c \mid \varphi) \cdot f_\varphi(\varphi) \cdot d\varphi \tag{13}$$

From the total p.d.f. $f_{c_k}(c)$, it is possible to determine the total mean,

$$\mu_{c_k} = \int_{-\infty}^{\infty} \mu_{c_k|\varphi} \cdot f_\varphi(\varphi) \cdot d\varphi \tag{14}$$

and variance of the number of counts:

$$\sigma_{c_k}^2 = \int_{-\infty}^{\infty} \sigma_{c_k|\varphi}^2 f_\varphi(\varphi) d\varphi + \int_{-\infty}^{\infty} \mu_{c_k|\varphi}^2 f_\varphi(\varphi) d\varphi - \left[\int_{-\infty}^{\infty} \mu_{c_k|\varphi} f_\varphi(\varphi) d\varphi \right]^2. \tag{15}$$

B. Asynchronous sampling

Considering now that the sampling is performed asynchronously with the stimulus signal, that is, the initial phase of the stimulus signal is not controlled. Let us consider that this initial phase can be taken as uniformly distributed between 0 and 2π. Furthermore, if the frequencies of the sampling clock and the stimulus signal are carefully chosen and are not harmonically related, then the ideal sample phases are uniformly spaced:

$$\gamma_j = j\frac{2\pi}{M} + \varphi . \tag{16}$$

With some manipulation an expression for the mean

$$\mu_{c_k} = \frac{M}{2\pi} \int_{-\pi}^{\pi} p_k(\gamma) d\gamma \tag{17}$$

and for the variance

$$\begin{aligned} \sigma_{c_k}^2 &= \mu_{\sigma_{c_k|\varphi}^2} + \sigma_{\mu_{c_k|\varphi}}^2 \\ \mu_{\sigma_{c_k|\varphi}^2} &= \frac{M}{2\pi} \int_{-\pi}^{\pi} p_k(\gamma)\left[1 - p_k(\gamma)\right] d\gamma \\ \sigma_{\mu_{c_k|\varphi}}^2 &= \frac{M}{2\pi} \int_0^{\frac{2\cdot\pi}{M}} \left(\sum_{j=0}^{M-1} p_k\left(j\frac{2\pi}{M} + \varphi \right) \right)^2 d\varphi - \left(\frac{M}{2\pi} \int_{-\pi}^{\pi} p_k(\gamma) d\gamma \right)^2 \end{aligned} \tag{18}$$

can be easily derived from (15), (16) and (17). These expressions are written in terms of the number of samples acquired (M) and the probability that a sample belongs to a class of the cumulative histogram (p_k). The value of this probability depends, in turn, on the normalized input-equivalent noise standard deviation (σ_n), on the normalized transition voltage ($U[k]$) and sample phase (γ_j).

The expression for the variance was divided into two terms: the mean of the conditional variance ($\mu_{\sigma_{c_k|\varphi}^2}$) and the variance of the conditional mean ($\sigma_{\mu_{c_k|\varphi}}^2$).

C. Mean of the conditional variance

In this paragraph we analyse the mean of the conditional variance. Inserting (10) into (18) leads to

$$\mu_{\sigma^2_{c_k|\varphi}} = \frac{M}{2\pi} \cdot \int_{-\pi}^{\pi} \frac{1}{4} \cdot \left[1 - erf^2 \left(\frac{U[k+1] + \text{tri}(\varphi)}{\sqrt{2} \cdot \sigma_n} \right) \right] \cdot d\varphi \qquad (19)$$

which is depicted in Fig. 2.

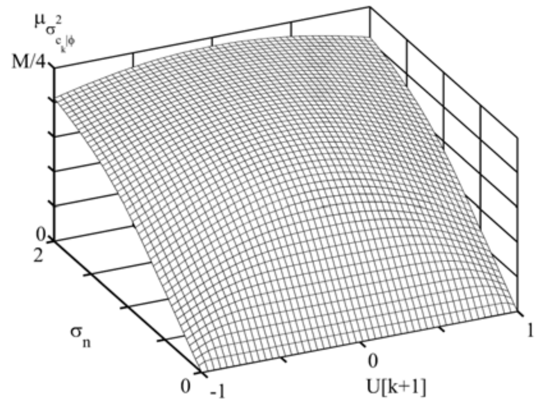

Fig. 2 Representation of the term $\mu_{\sigma^2_{c_k|\varphi}}$ as a function of the standard deviation of the input-equivalent noise (σ_n) and of the normalized transition voltage.

The term $\mu_{\sigma^2_{c_k|\varphi}}$ of the variance of the number of count of the histogram is maximum for a transition voltage equal to the stimulus signal offset ($U[k]$=0). Ideally this would be the middle of the ADC input range. In this case the dependence of $\mu_{\sigma^2_{c_k|\varphi}}$ on the standard deviations of input-equivalent noise is represented in Fig. 3.

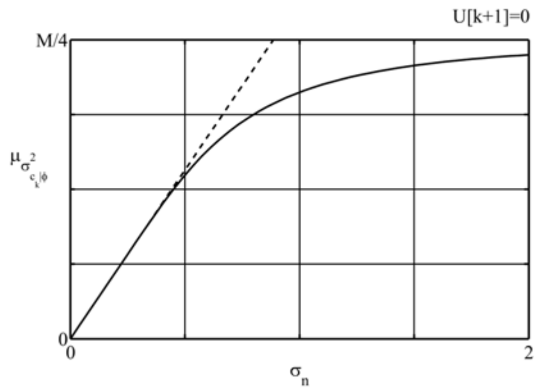

Fig. 3 Representation of the term $\mu_{\sigma^2_{c_k|\varphi}}$ as a function of the standard deviation of the input-equivalent noise (σ_n) for a null value of normalized transition voltage. The dotted line represents the approximation for small values of input-equivalent noise standard deviation given by (27).

As the input-equivalent noise standard deviation goes to 0, the mean of the conditional variance is directly proportional to that same standard deviation. To prove that we will compute the following limit:

$$\lim_{\sigma_n \to 0} \frac{\mu_{\sigma^2_{c_k|\varphi}}}{\sigma_n} = \frac{M}{2\pi\sqrt{\pi}} \lim_{\sigma_n \to 0} \int_{-\pi}^{\pi} \frac{\sqrt{\pi}}{4 \cdot \sigma_n} \cdot \left[1 - erf^2 \left(\frac{U[k+1] + \text{tri}(\varphi)}{\sqrt{2} \cdot \sigma_n} \right) \right] d\varphi \qquad (20)$$

Introducing now variable *x*:

$$x = U[k+1] + \text{tri}(\varphi) \qquad (21)$$

and inserting it into (20), leads to

$$\lim_{\sigma_n \to 0} \frac{\mu_{\sigma^2_{c_k|\varphi}}}{\sigma_n} = \frac{M}{\pi\sqrt{\pi}} \lim_{\sigma_n \to 0} \int_{U[k+1]-1}^{U[k+1]+1} \frac{\sqrt{\pi}}{4 \cdot \sigma_n} \frac{\pi}{2} \left[1 - erf^2 \left(\frac{x}{\sqrt{2} \cdot \sigma_n} \right) \right] \cdot dx \qquad (22)$$

To obtain (22) we rewrote (21) as

$$\varphi = atri\left(x - U\left[k+1\right]\right). \tag{23}$$

The derivative $\partial\varphi/\partial x = \pi/2$.

The limits of the integral in (22) can be extended to infinite for small values of σ_n:

$$\lim_{\sigma_n \to 0} \frac{\mu_{\sigma^2_{c_k|\varphi}}}{\sigma_n} = \frac{M}{\pi\sqrt{\pi}} \lim_{\sigma_n \to 0} \int_{-\infty}^{\infty} \frac{\sqrt{\pi}}{4 \cdot \sigma_n} \frac{\pi}{2} \left[1 - erf^2\left(\frac{x}{\sqrt{2} \cdot \sigma_n}\right)\right] \cdot dx. \tag{24}$$

Considering that [12]

$$\lim_{\sigma_n \to 0} \int_{-\infty}^{\infty} \frac{\sqrt{\pi}}{4 \cdot \sigma_n} \left[1 - erf^2\left(\frac{x}{\sqrt{2} \cdot \sigma_n}\right)\right] \cdot dx = 1 \tag{25}$$

we have

$$\lim_{\sigma_n \to 0} \frac{\mu_{\sigma^2_{c_k|\varphi}}}{\sigma_n} = \frac{M}{2\sqrt{\pi}} \tag{26}$$

which demonstrates that we can approximate the mean of the conditional variance by

$$\mu_{\sigma^2_{c_k|\varphi}} = \frac{M}{2\sqrt{\pi}} \sigma_n. \tag{27}$$

For large values of standard deviation the term $\mu_{\sigma^2_{c_k|\varphi}}$ approaches M/4. We can thus write an expression that gives us the upper limit of the term $\mu_{\sigma^2_{c_k|\varphi}}$:

$$\mu_{\sigma^2_{c_k|\varphi}} \le M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{2\sqrt{\pi}}\right). \tag{28}$$

D. Variance of the conditional mean

The dependence of the term $\sigma^2_{\mu_{c_k|\varphi}}$ on the number of samples and the transition voltage is less monotone then the term $\mu_{\sigma^2_{c_k|\varphi}}$ (Fig. 4).

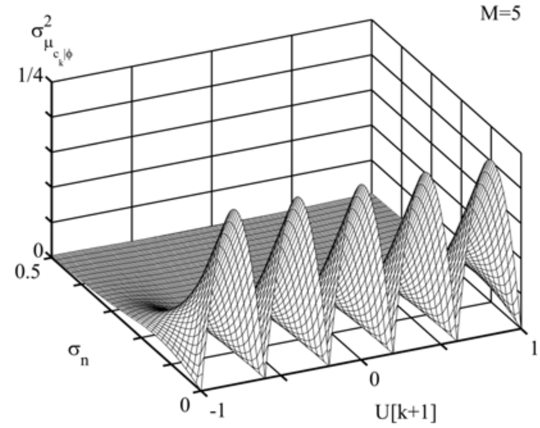

Fig. 4 Representation of the term $\sigma^2_{\mu_{c_k|\varphi}}$ as a function of the normalized transition voltage ($U[k+1]$) and the input equivalent noise (σ_n).

For small values of the standard deviation of the noise the term $\sigma^2_{\mu_{c_k|\varphi}}$ depends strongly on the transition voltage. The number of arcs seen in Fig. 4 is equal to the number of samples.

The maximum value of this term occurs in the absence of noise and it is equal to ¼.

E. Variance of the number of counts

The variance of the number of counts is determined by adding the terms $\mu_{\sigma^2_{c_k|\varphi}}$ and $\sigma^2_{\mu_{c_k|\varphi}}$ (Fig. 5).

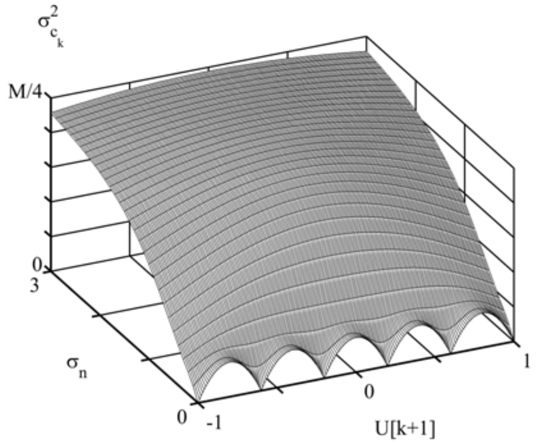

Fig. 5 Representation of the term $\sigma^2_{c_k}$ as a function of the normalized transition voltage and of the standard deviation of the input-equivalent noise for the case of M = 5.

Combining the conclusions from the two previous sections we reach the following expression for the upper limit of the variance of the number of counts.

$$\sigma^2_{c_k} \le \max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{2\sqrt{\pi}}\right)\right). \tag{29}$$

F. Variance of the Transition Voltages

The determination of the variance of the estimated transition voltages is, using (1) [11]:

$$\sigma^2_{T_k} = \left(\frac{2A}{M}\right)^2 \sigma^2_{c_k} \tag{30}$$

Inserting (29) leads to

$$\sigma^2_{T_k} \le \left(\frac{2A}{M}\right)^2 \max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{2\sqrt{\pi}}\right)\right) \tag{31}$$

IV. Variance of the Code Bin Widths

A. Probability of a given sample belonging to a bin of the histogram

The code bin widths are computed from the histogram using (2). The analysis of the variance of the number of counts of the histogram is similar than the one presented for the cumulative histogram. The first difference is that the probability of a given sample belonging to a bin k of the histogram is given by

$$\begin{aligned} p_k(\gamma_j) &= \int_{U[k]}^{U[k+1]} f_{u_j}(u \mid \gamma_j) \cdot du = \\ &= \frac{1}{2} erf\left(\frac{U[k+1] + tri(\gamma_j)}{\sqrt{2} \cdot \sigma_n}\right) - \frac{1}{2} erf\left(\frac{U[k] + tri(\gamma_j)}{\sqrt{2} \cdot \sigma_n}\right) \end{aligned} \tag{32}$$

instead of (10). This probability is depicted in Fig. 6 as a function of sample phase, γ_j and input-equivalent noise standard deviation, σ_n.

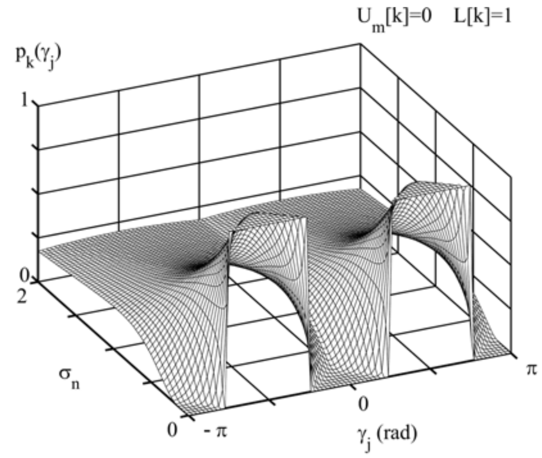

Fig. 6 Representation of the probability that a sample belongs to a class of the histogram as a function of the sample phase (γ_j) and of the input-equivalent noise standard deviation (σ_n). Example with $U[k+1]=0$ (transition voltage equal to the DC value of the stimulus signal).

Note that instead of using $U[k]$ and $U[k+1]$ it is easier to use the mean value of the normalized transition voltages and the code bin width given, in normalized units, by $U_m[k]$ and $L[k]$:

$$U_m[k] = \frac{U[k+1]+U[k]}{2} \quad , \quad L[k] = U[k+1] - U[k] \,. \tag{33}$$

B. Mean of the conditional variance

The mean of the conditional variance of the number of counts of the histogram is obtained using (18) and (32). The result, as a function of the input-equivalent noise standard deviation and mean normalized transition voltage is depicted in Fig. 7.

In Fig. 8 the mean of the conditional variance is depicted as a function of the input-equivalent noise for a null mean transition voltage and a normalized code bin width of 0.02 (2% of the stimulus signal amplitude).

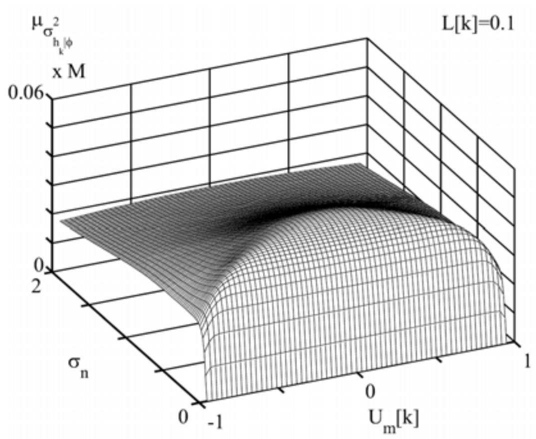

Fig. 7 Representation of the term $\mu_{\sigma^2_{h_k|\varphi}}$ as a function of the standard deviation of the input-equivalent noise (σ_n) and of the normalized transition voltage.

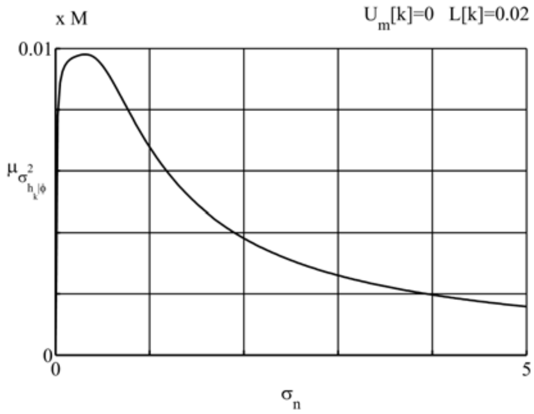

Fig. 8 Representation of the term $\mu_{\sigma^2_{c_k|\varphi}}$ as a function of the standard deviation of the input-equivalent noise (σ_n).

Fig. 9 depicts the same term for small values of noise standard deviation.

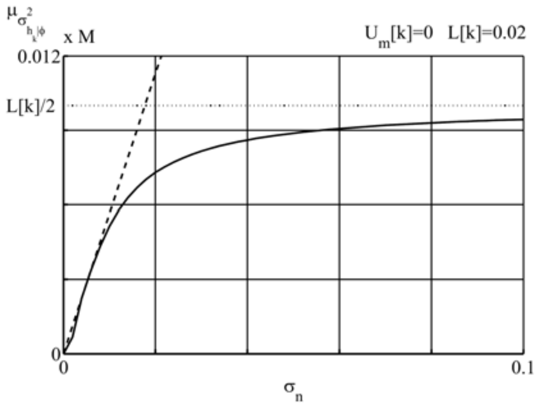

Fig. 9 Representation of the term $\mu_{\sigma^2_{h_k|\varphi}}$ as a function of the standard deviation of the input-equivalent noise (σ_n). The dashed line represnts the approximation for low values of noise standard deviation given by (34). The dotted line represents the maximum values attained by the mean of the conditional variance.

It can be seen that for small values of noise standard deviation the behavior of the mean of the conditional variance approaches a straight line. To compute the slope of this straight line recall that the number of counts of bin *k* of the histogram is the difference between the number of counts in bin *k*+1 and *k* of the cumulative histogram as expressed in (3). For low values of noise standard deviation the transition voltages (and the number of counts in the cumulative histogram) become uncorrelated. In this case the variance of the number of counts of the histogram is just twice the variance of the number of counts of the cumulative histogram, given by (27):

$$\mu_{\sigma^2_{h_k|\varphi}} = \frac{M}{\sqrt{\pi}} \cdot \sigma_n \,. \qquad (34)$$

This equation represents the dashed line in Fig. 9.
In Fig. 8 and Fig. 9 we can see that the mean of the conditional variance has a maximum of L[*k*]/2.

C. Variance of the conditional mean

The variance of the conditional mean of the histogram (Fig. 10) has a similar behavior than it had in the case of the cumulative histogram (Fig. 4).

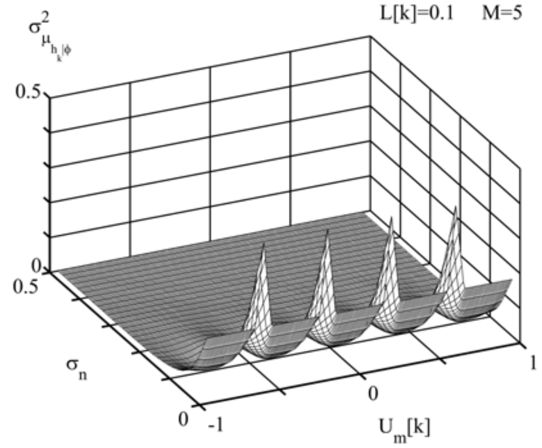

Fig. 10 Representation of the term $\sigma^2_{\mu_{h_k|\varphi}}$ as a function of the normalized transition voltage ($U[k+1]$) and the input equivalent noise (σ_n).

In the case of the cumulative histogram, for small values of noise standard deviation, the variance of the conditional mean approached ¼. In this situation the number of counts in the cumulative histogram are uncorrelated and the variance of the number of count of the histogram will be twice that of the cumulative histogram, that is, ½.

D. Variance of the number of counts

Using the conclusion that where reached in B and C we suggest the following expression for the upper limit of the number of counts of the histogram:

$$\sigma^2_{h_k} \le \max\left(\frac{1}{2}, \min\left(\frac{L[k]}{2}, \frac{\sigma_n}{\sqrt{\pi}} M\right)\right). \tag{35}$$

E. Variance of the Code Bin Widths

The values of the code bin widths can be obtained directly from the number of counts of the histogram. The variance of the code bin widths, considering (2), can thus be calculated by [11]

$$\sigma^2_W = \left(\frac{2A}{M}\right)^2 \sigma^2_{h_k} . \tag{36}$$

Substituting (35) in (36) leads to

$$\sigma^2_W \le \left(\frac{2A}{M}\right)^2 \max\left(\frac{1}{2}, \min\left(\frac{L[k]}{2}, \frac{\sigma_n}{\sqrt{\pi}} M\right)\right). \tag{37}$$

V. Conclusion

In this paper we derived mathematical expressions for the determination of the precision of the estimates of transition voltages and code bin widths of an ADC tested with the Histogram Test Method using triangular stimulus signals. This expressions account for the effect of input-equivalent random noise on the test setup and on the ADC itself. They are used to compute an uncertainty interval for the estimates made which follows modern measurements practices like the ones recommended in GUM [13] and whose intent is to express the quality of measurements.
Since the histogram procedure is a statistical one where a large number of samples are acquired, it is important to know the minimum number of samples necessary in order to minimize the test time. This can be done using the expressions presented here.

Acknowledgments

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