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Fundamental and Applied Metrology
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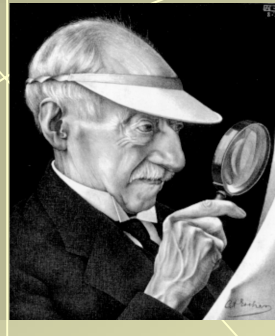

Round Table on VIM: Measurement vs. Mathematical Modelling

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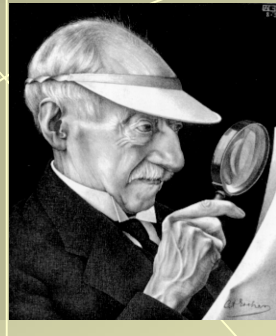
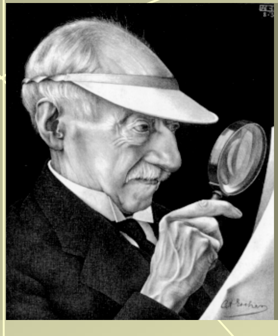
STATEMENT TO BE CONSIDERED



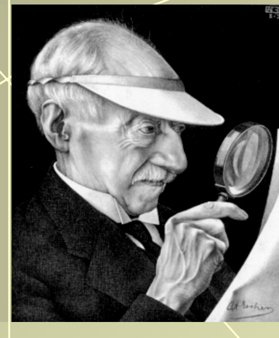
There is a need for deeper and more consistent linking of the concepts of measurement science with the concepts of mathematical modelling

- The concept of mathematical model is denoted in VIM3 with the term "model" (*cf.* 2.6, 2.31–33, 2.41, 2.48–2.51)
- The mathematical model of a system is its description – composed of entities such as numbers, variables, sets, equations, functions and relations – which enables one to infer about the properties and/or the behaviour of that system under various conditions
- The process leading to the formulation of a mathematical model is an iterative procedure composed of two fundamental operations:
  - *structural identification* (selection of a structure for the model)
  - *parametric identification* (estimation of the model parameters)

OUTLINE OF JUSTIFICATION

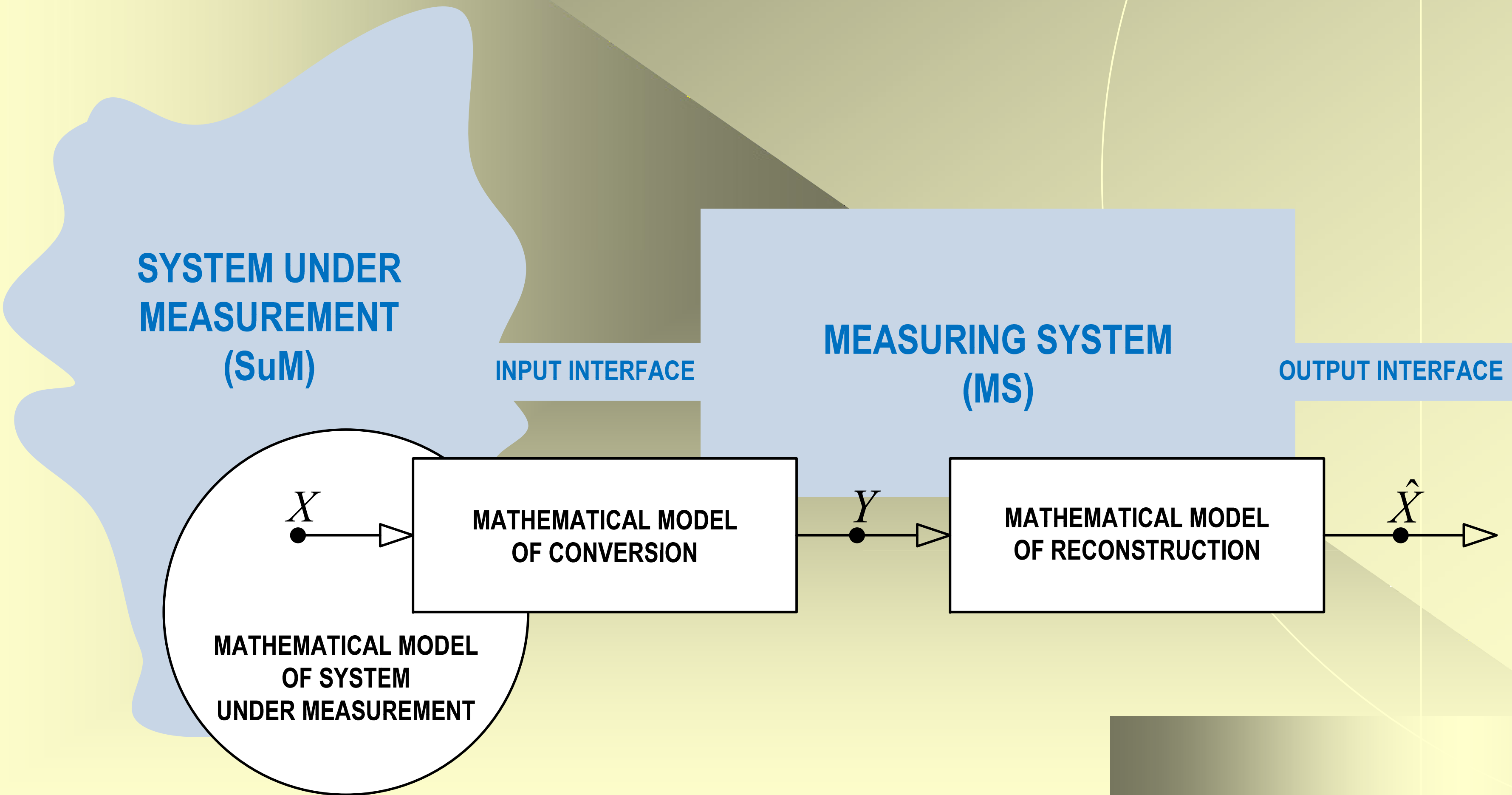


- Mathematical modelling is of primary importance for science (John von Neumann: *"...science does not attempt to explain, almost does not attempt to interpret; first of all, science is involved in creation of mathematical models"*)
- Mathematical modelling is of primary importance for science-based technology
- Measurement may be viewed as (partial) identification of a mathematical model of SuM
- Mathematical models of physical and (bio)chemical objects cannot be identified without measurements.

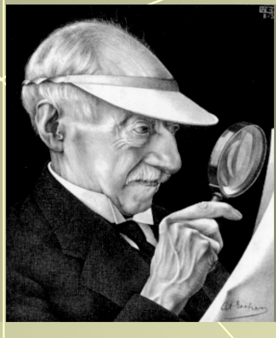


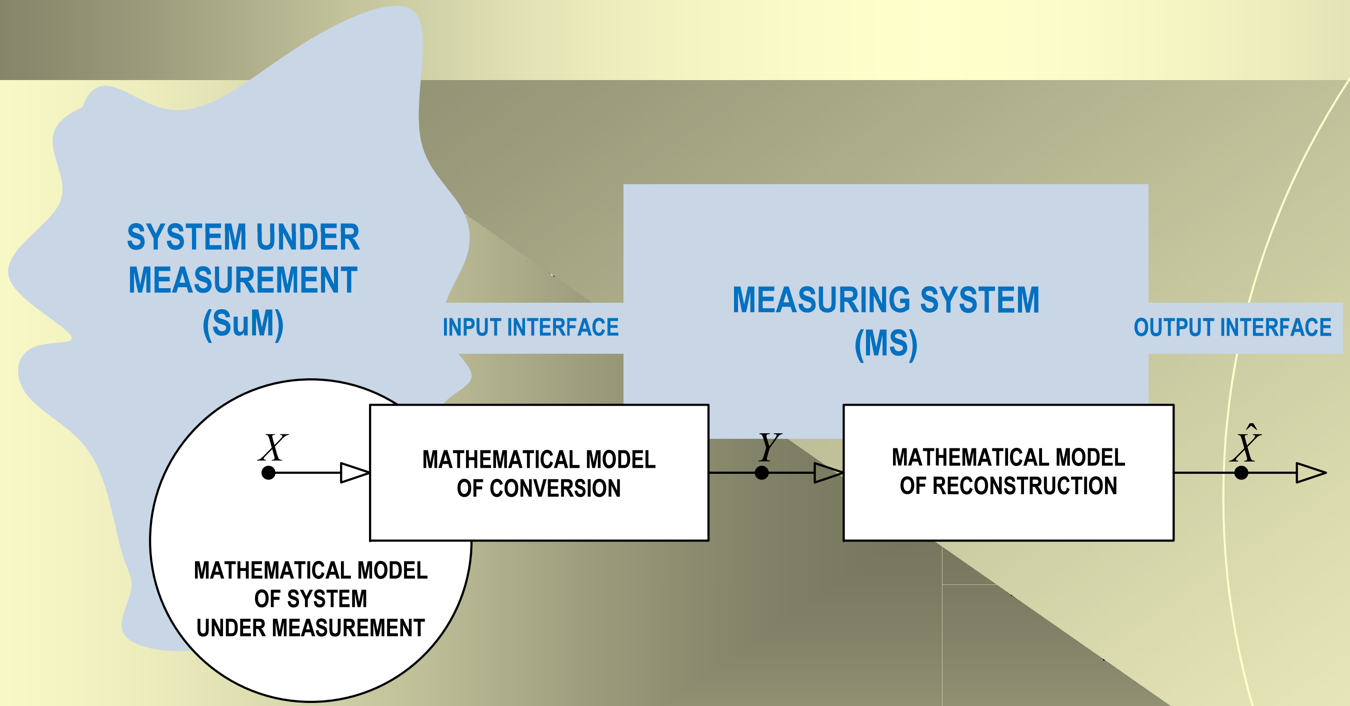
HYPOTHETICAL SOLUTION

- The *input-output models are* of particular practical importance for R&D
- A useful meta-model of measurement may be composed of three *input-output models*:



HYPOTHETICAL SOLUTION

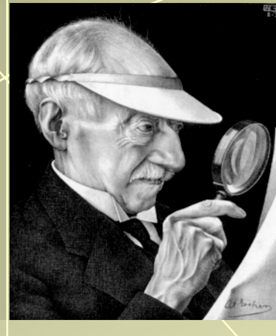


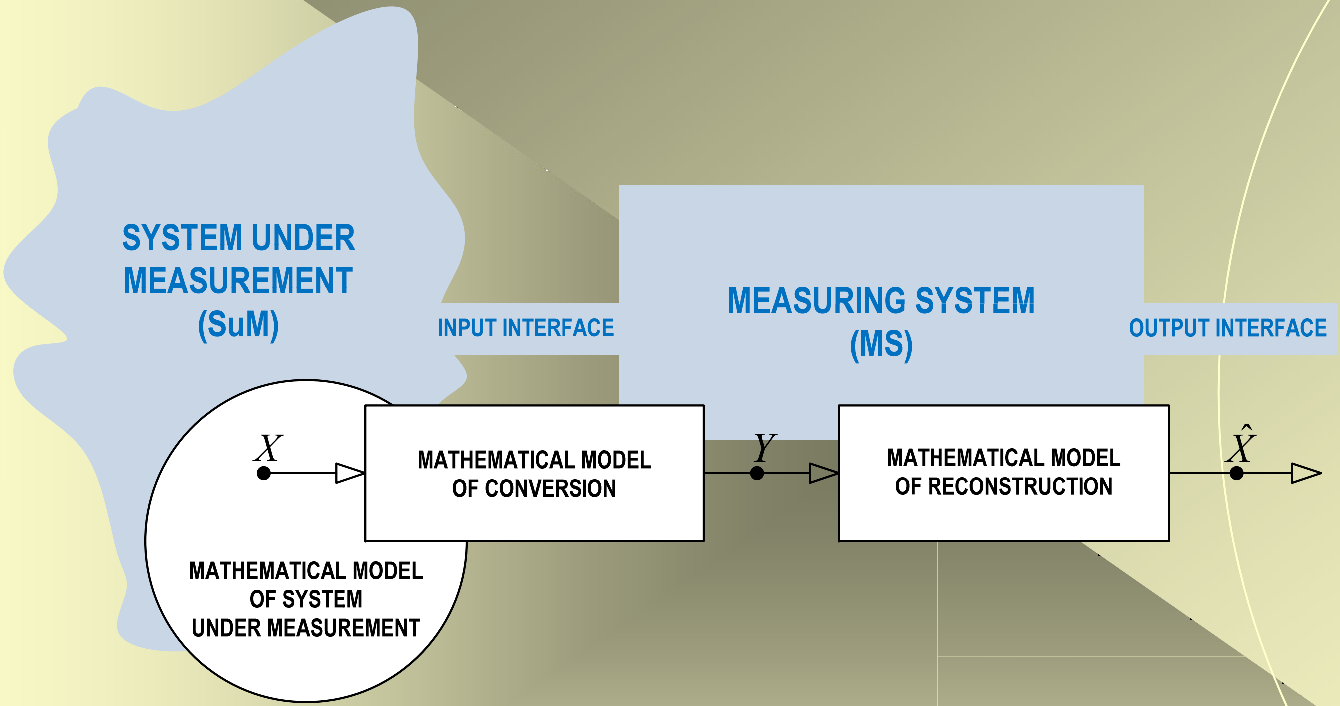


- A ***measurand*** X is defined by a mathematical model of SuM as a parameter or a function explicitly appearing in this model, or a function of such parameters or functions, or a functional of such parameters or functions

- A mathematical model of ***reconstruction*** is describing all the operations aimed at determination of the final result of measurement, *i.e.* an estimate of X, on the basis of the raw result of measurement Y, the mathematical model of conversion, and available *a priori* information on the measurand

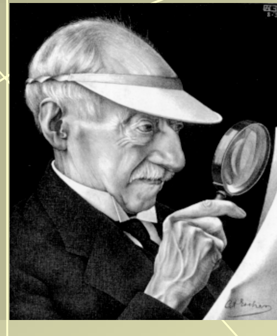
HYPOTHETICAL SOLUTION

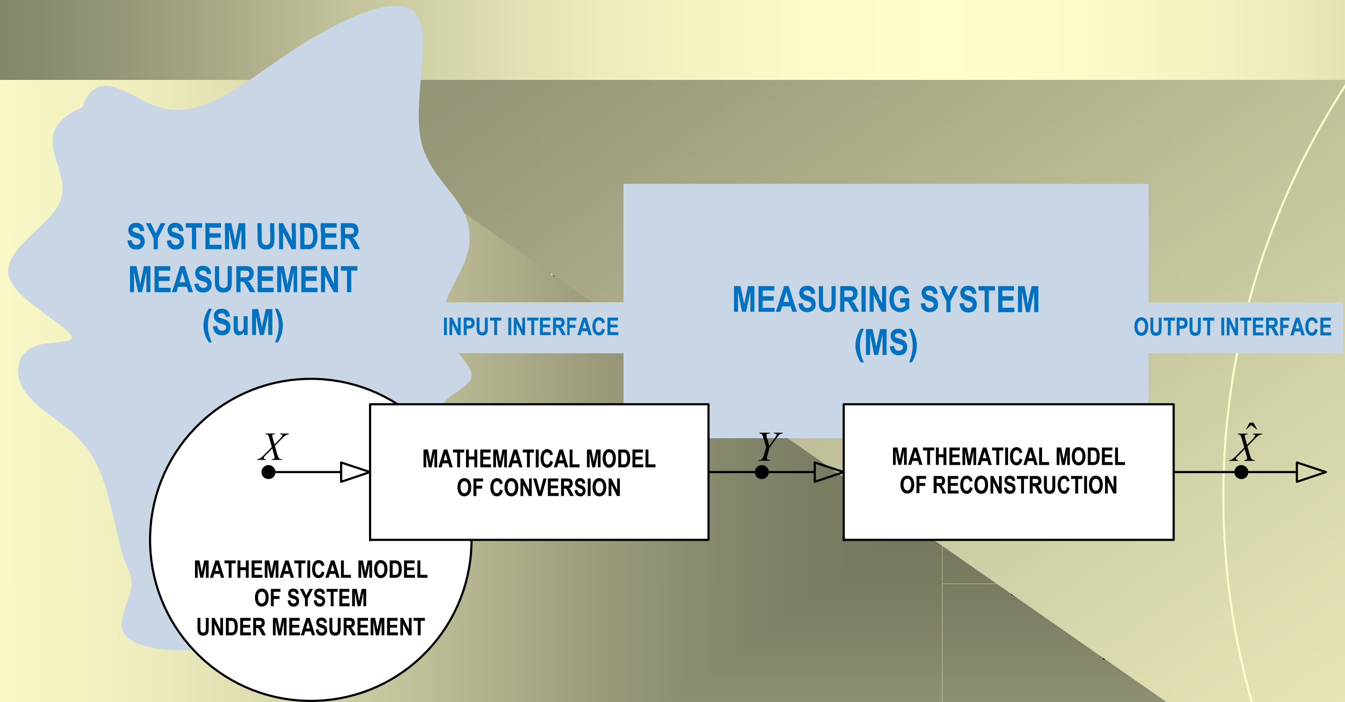




- The parameters of the forward or (pseudo)inverse model of conversion, necessary for measurand reconstruction, are obtained during ***calibration*** on the basis of the known (assumed) structure of the model and raw results of measurements corresponding to some reference values of the measurand

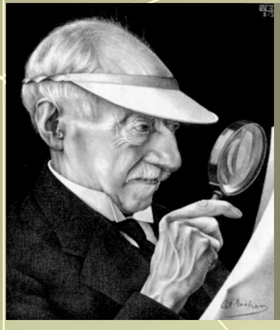
HYPOTHETICAL SOLUTION





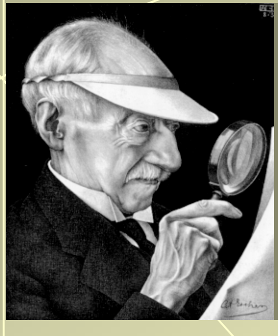
- The final result of measurement is subject to uncertainty resulting from:
  - the discrepancy between the mathematical models (white blocks) and physical reality (blue-grey blocks)
  - the discrepancy between and resulting from the properties of the mathematical models
- The evaluation of the first component is based on the assessment of the discrepancy between two mathematical models of the measurement channel: the principal model and the corresponding extended model

ADVANTAGES OF HYPOTHETICAL SOLUTION



- The presented approach is adequate and productive both in simple cases (analogue voltmeters) and in very complex cases (hyperspectral analysers)
- It provides an elegant and efficient language for the description of measurement when the measurand (and other quantities involved) is a vectorial function of time, space coordinates and/or other quantities
- It provides a logical basis for a consistent definition of measurand and of calibration

CREDENTIALS



- The author's direct positive experience with the proposed solution is related to:
  - dynamic calorimeters
  - ultrasonic sensors of alcohol concentration
  - fibre-optic sensors of high pressure, temperature, vibration
  - spectrophotometric transducers
  - absorption spectrophotometers
  - optical performance monitors for DWDM telecommunication systems
  - spectrophotometric analysers of food

- The author's indirect positive experience is reported in two review papers:
  - R. Z. Morawski: "Sensor Applications of Digital Signal Processing", in: *Encyclopedia of Sensors* (Eds. C. A. Grimes, E. C. Dickey, M. V. Pishko), American Scientific Publisher, 2006, Vol. 9, pp. 135–164
  - R. Z. Morawski: "Spectrophotometric Applications of Digital Signal Processing", *Measurement Science and Technology*, Vol. 17, August 2006, pp. R117–R144